CHAPTER 11

RESONANCE & FILTERS

When I introduce the concept of impedance I often point out that the relationship between voltage and current is frequency dependent. This complicates analysis and can lead to undesirable effects. However, this frequency dependence also allows us to design circuits that discriminate between wanted and unwanted signals based on frequency. Whether the frequency response of the circuit is designed for a purpose or it emerges due to the fabrication and assembly methods of the individual components, we have to characterize and describe the behavior.

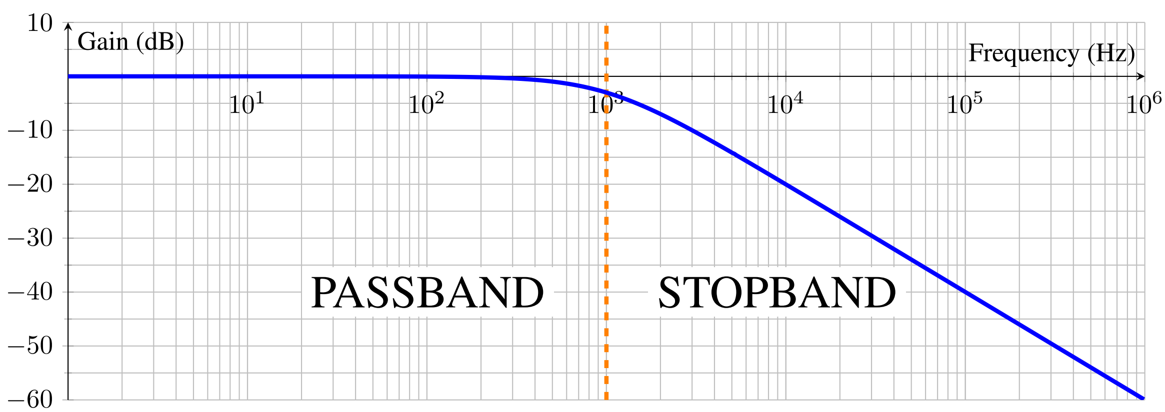
11.1 Pass-bands and Stop-bands

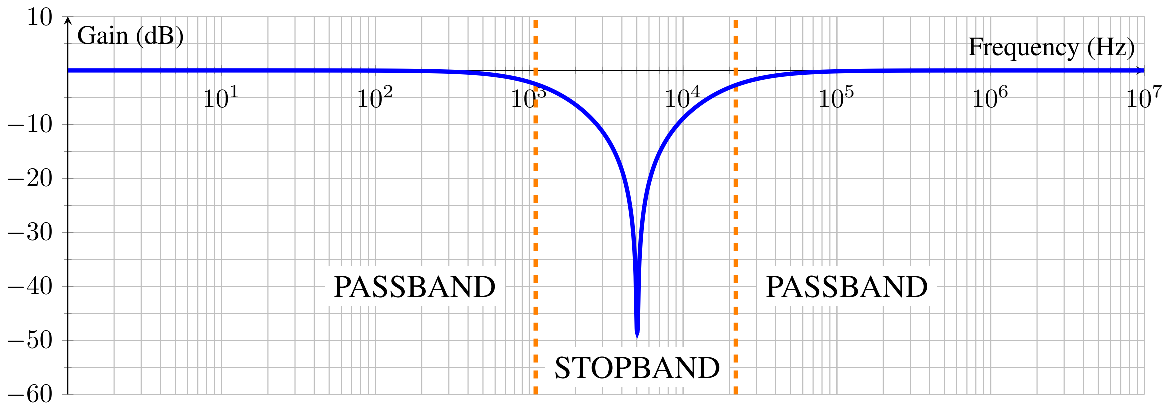
Filters discriminate between frequencies. Some signals will pass through a filter. Some will be stopped by a filter. We refer to the ranges of frequencies that are passed and stopped as the pass-band and stop-band of the filter.

Pass-band: The range of frequencies that **pass** through a system

Stop-band: The range of frequencies that are **stopped** by a system

Visually we can see these bands in the magnitude response on a Bode plot. Here are two examples of these bands shown on two different filter responses.



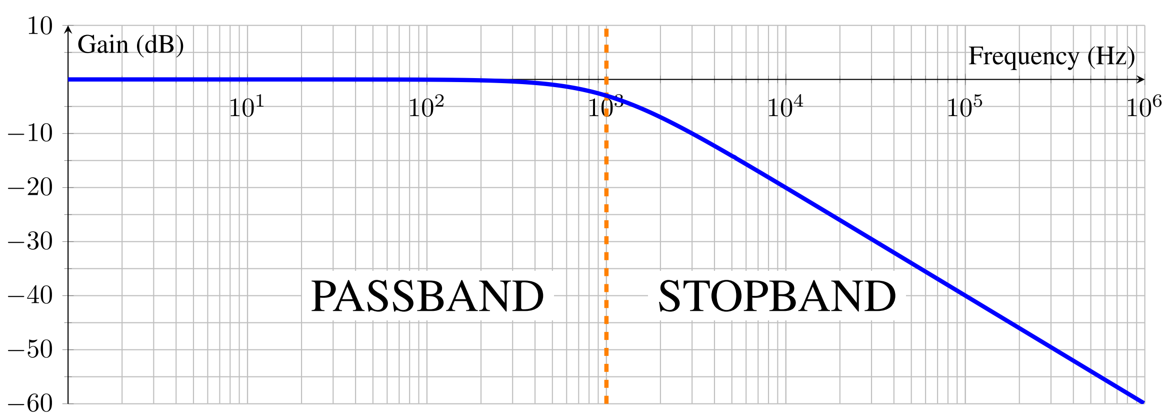


11.2 Types of Filters

The frequency response of a circuit can take on any shape. There are, however, some frequency responses that are given names as they are quite common. The first four listed here are the most common. I will show the magnitude response for each and give a brief description.

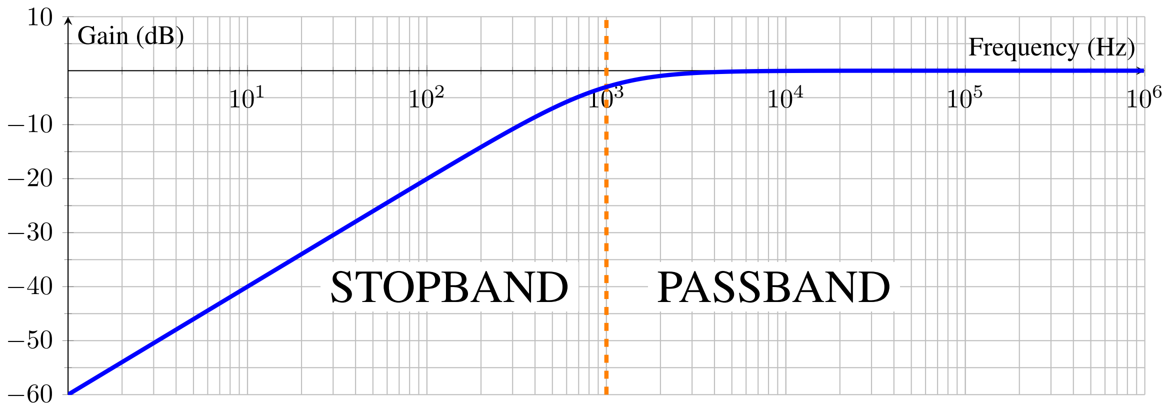
11.2.1 Low-pass

A low-pass filter allows low frequencies to pass. Above the single cut-off frequency the signal is attenuated (or stopped).



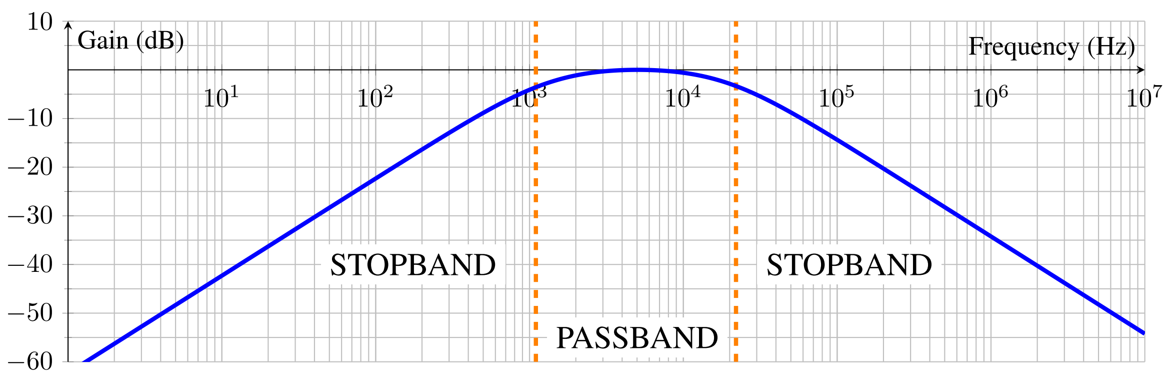
11.2.2 High-pass

A high-pass filter allows high frequencies to pass. Below the single cut-off frequency the signal is attenuated (or stopped).



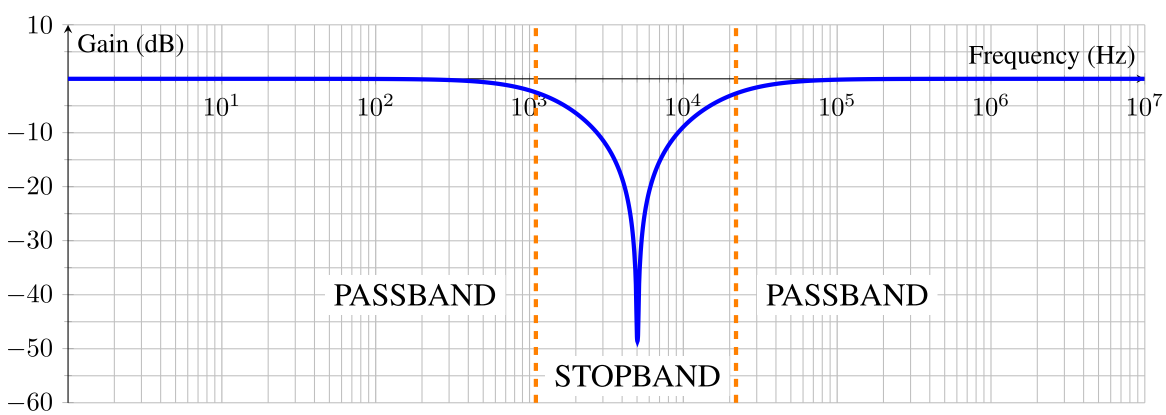
11.2.3 Band-pass

A band-pass filter passes the signal between two cut-off frequencies. Below and above those cut-off frequencies the filter attenuates the signal.



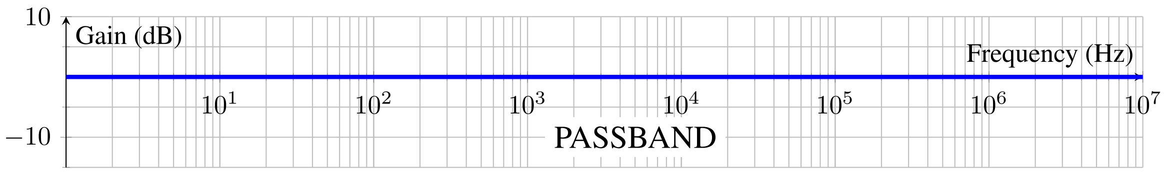
11.2.4 Notch, Stop-band

A notch (or stop-band) filter attenuates the signal between two cut-off frequencies. Below and above those cut-off frequencies the signal passes through the filter.

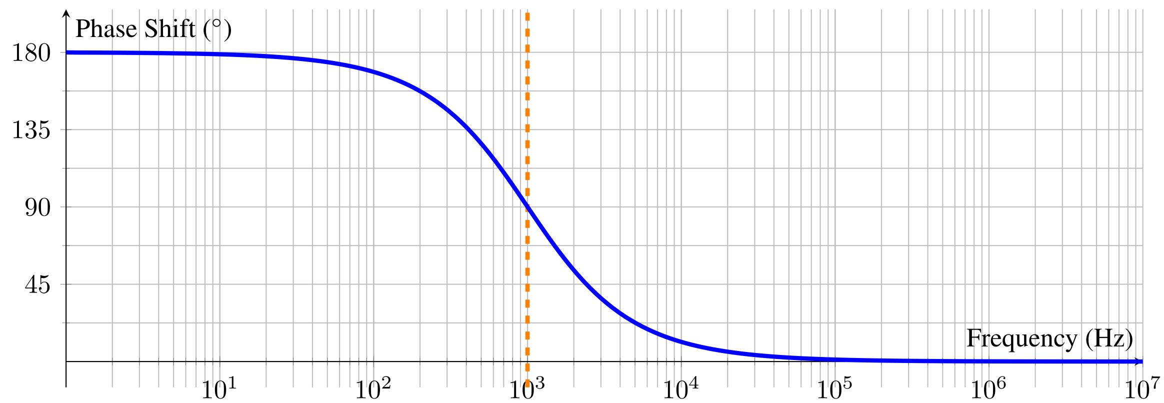


11.2.5 All-pass, Phase-shift

Now let's look at a less common, but still named, frequency response, the all-pass filter. This filter is used to create muscial effects and to stabilize otherwise unstable, multi-stage filters. The magnitude response is even across all frequencies. That is, it is unity as a scalar gain or 0 dB. The magnitude response is shown here though it should be quite boring.



We can find the interesting response of an all-pass filter in its phase-shift response.



You may notice that there are no cut-off frequencies, center frequencies, etc. to characterize this filter's response. Therefore, we usually discuss the filter's characteristic based on the phase-shift response. When describing the filter's response we state the frequency at which the phase-shift passes through $\pm 90°$.

11.3 Critical Frequencies

All of the filters shown above have borders between the pass and stop bands. Some of the filters have multiple borders. We can locate these borders by finding the critical frequencies of the filter on the magnitude response and/or the phase-shift response.

Cut-off/-3 dB Frequencies The first group of critical frequencies define the borders between pass-bands and stop-bands. There are a myriad of names for these frequencies including cut-off, half-power, and -3 dB frequencies. For the common filter types introduced above there are between zero and two cut-off frequencies. We'll discuss the conditions that lead to these frequencies later on in this chapter.

When there is one critical frequency it is simply referred to as "the" cut-off frequency. When there are two critical frequencies we differentiate between the two by calling one the "Lower cut-off frequency" and the other the "Upper cut-off frequency". We notate a single cut-off frequency using f_c. The lower and upper cut-off frequencies are commonly notated as f_1 and f_2.

Center/Resonant Frequencies Resonant frequencies define another class of critical frequencies. Like cut-off frequencies, there may be zero, one, or more resonant frequencies depending on the construction of the circuit. We'll talk about what leads to resonance in a circuit later in this chapter. Not all circuits resonate. Of the filter circuits introduced above, only the pass-band filter and the stop-band filter have a resonant frequency. This frequency defines the center of the pass and stop bands of those filters. We notate the resonant frequency as f_r.

11.4 Filter Order

What defines filter order?

- number and type of components
- rate of change (roll-off) between pass and stop bands
- Transfer function orders

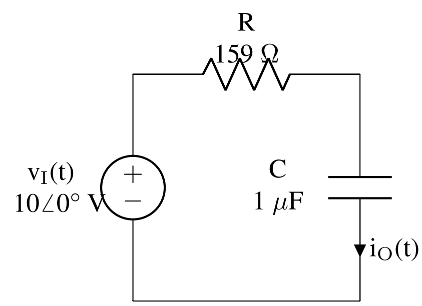
11.5 First Order Filters

Low-pass or high-pass

11.5.1 Considering Power

- Supplied vs Dissipated
- RC Circuit, look at power dissipated by the resistor
- Look at the current
- look at the impedance seen by the source

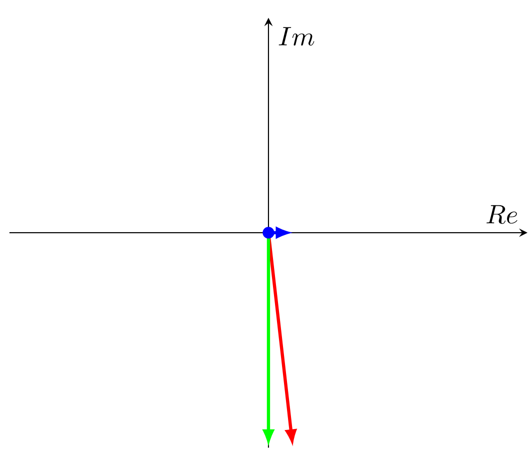
11.5.2 RC Filter



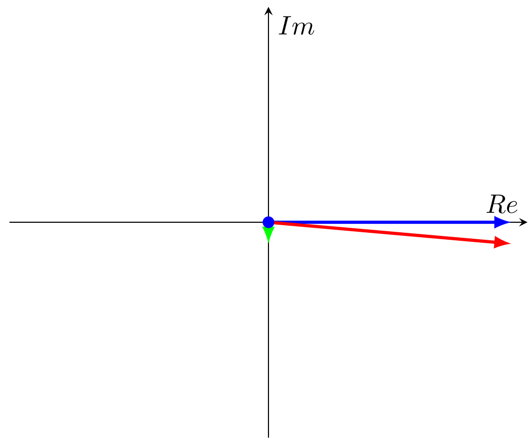
$$H(s) = \frac{Cs}{RCs + 1}$$

Analysis RC circuit 10 volt cos RC ckt 159 ohm/1microFarad

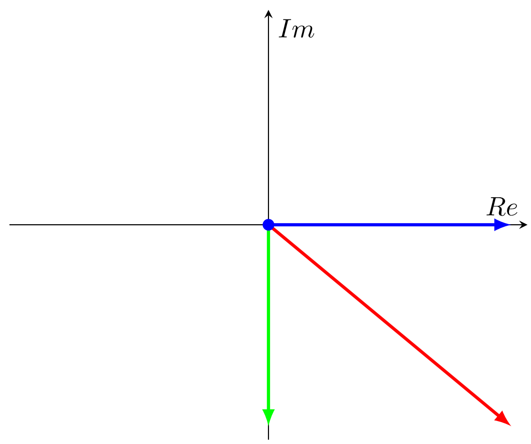
at 100 Hz calculate Zc, Zt,I point out magnitude of R vs Zc (looks more capacitive) highlight mag and angle of I



at 10kHz calculate Zc, Zt,I point out magnitude of R vs Zc (looks more resistive) highlight mag and angle of I



at w_c tau=RC w_c=1/tau f_c=w_c/(2*pi) calculate Zc, Zt,I draw phasor diagram point out Zc mag equals R (It's not resistive or capacitive, it's a balance between the two)



f	R	Zc	Z	I_o	A (V_O)	A ($_O$)
100 Hz	150 Ω	-j1.59 kΩ	150-j1.59 kΩ	6.26∠84.6° mA	1	
10k Hz	150 Ω	-j15.92 Ω	150-j15.92 Ω	66.3∠6.1° mA	0	
1.06k Hz	150 Ω	-j150 Ω	150-j150 Ω	47.14∠45° mA	0.707	

Bode plot with frequencies and bands labeled

11.5.3 Things That Define a First-order Filter

1. Filter-type
2. Cut-off frequency
3. Pass-band Gain

11.6 Second Order Filters

Bandwidth: The distance between cut-off frequencies. The width of the pass-band.
Band-stop vs Band-pass Also... 2nd order high vs. low pass

11.6.1 Resonance

11.6.2 RLC Filter

Calc Zl, Zc, Zt, I for five frequencies

- 10 volts,
- Point out mag of R vs Zc+Zl
- Highlight mag/angle of I
- Draw Phasor diagram for each

Show Bode plot, point out frequencies and bands

11.6.3 Things That Define a Second-order Filter

1. Filter Type
2. Pass-band Gain
3. Center-frequency
4. Q-factor

11.6.4 Example of second order low/high pass filters

11.7 Higher-order Filters

11.8 Poles and Zeros

1. Num/Den Roots
2. Pole/Zero Diagrams
3. Examples

11.9 Approximating Bode Plots

- Use the poles/zeros to find critical frequencies
- Work left to right
- Find DC Gain
  - Example with finite DC Gain
  - Example that has infinite gain
- Decide where the bode plot will start (lowest frequency)
- Find the Gain at that frequency
- Find poles/zeros
- Zeros: +20dB/DEC
- Poles: -20dB/DEC
- work left to right building approximation
- plot approximation
- plot approximation vs calculated bode plot
- approximate bandwidth from poles/zeros
  - example that works (low Q)

- example that doesn't work (High Q)
- Oh crap, how do we handle this
    * Simple second order filters
    * General Case

$$H(s) = \frac{1}{RCs + 1}$$

R=1 kΩ C=159 nF

```
clear all
close all
clc
format short eng

syms s
N=[1];  %coefficients of the numerator
D=[1e3*159e-9 1]; %coefficients of the denominator
zeros=abs(roots(N)); %rad/s
poles=abs(roots(D)); %rad/s

zeros/(2*pi) %convert to Hz
poles/(2*pi) %convert to Hz
```

$$H(s) = \frac{\frac{L}{R}s}{\frac{L}{R}s + 1}$$

R=628 Ω L=100 mH

```
clear all
close all
clc
format short eng

syms s
N=[100e-3/628 0];  %coefficients of the numerator
D=[100e-3/628 1]; %coefficients of the denominator
zeros=abs(roots(N)); %rad/s
poles=abs(roots(D)); %rad/s

zeros/(2*pi) %convert to Hz
poles/(2*pi) %convert to Hz
```

$$H(s) = \frac{10000}{0.012s^2 + 1.42s + 10000}$$

```
clear all
close all
clc
format short eng

syms s
N=[10000];  %coefficients of the numerator
D=[.012 1.42 10000]; %coefficients of the denominator
zeros=abs(roots(N)); %rad/s
poles=abs(roots(D)); %rad/s

zeros/(2*pi) %convert to Hz
poles/(2*pi) %convert to Hz
```

11.10 Another Way to Type a Filter

11.10.1 Passive

11.10.2 Active

11.10.3 Switched-Capacitor

11.11 Other Filter Attributes

11.11.1 DC Gain

11.11.2 Ripple

11.11.3 Transient Response

11.11.4 Accuracy

11.11.5 Cost

11.11.6 Circuit Size

11.11.7 Noise

11.11.8 Tunabillity

11.11.9 Frequency Range

11.11.10 Aliasing

11.11.11 Offset Voltage

11.11.12 Ringing

11.11.13 Monotonicity

11.12 Common Higher-Order Filter Types

11.12.1 Butterworth

11.12.2 Chebyshev

11.12.3 Bessel

11.12.4 Elliptic

11.13 Designing a Filter