CHAPTER 9

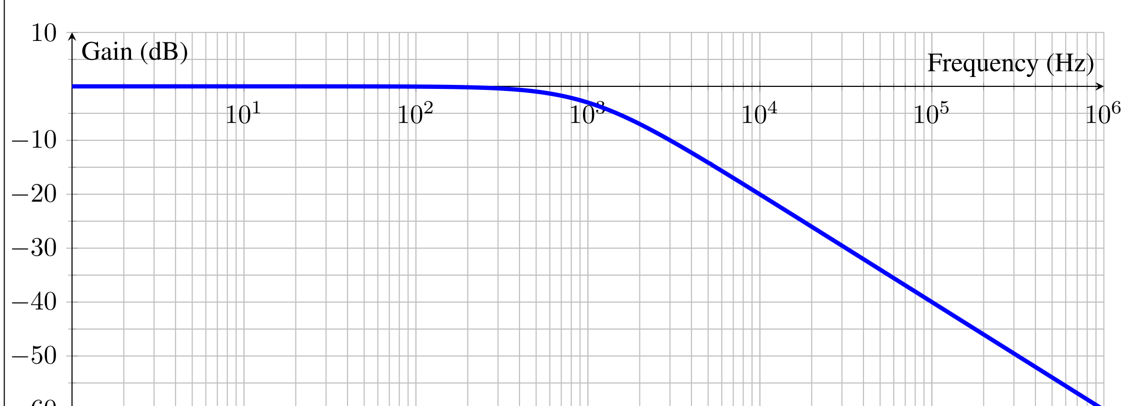
BODE PLOTS

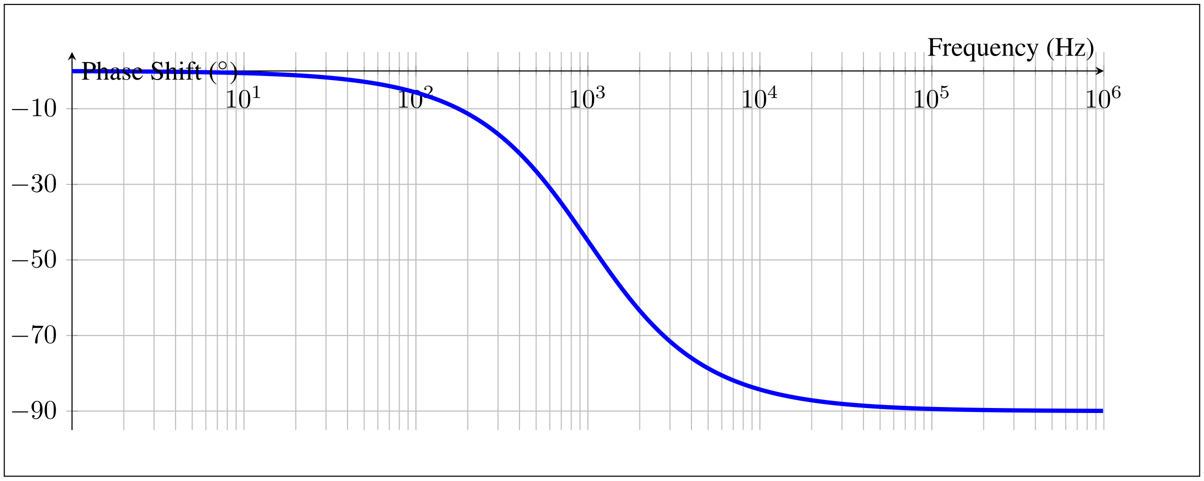
Now that we understand complex gain and decibels, we are ready to examine another means of communicating the complex gain of a circuit. In a previous chapter we found that complex gain varies with frequency. We did this by calculating complex gain for individual frequencies. While we could build a table of these values it would be difficult to visualize the circuits behavior over many decades of frequencies. A Bode plot is simply a plot of the two components of complex gain (magnitude and phase shift) over several decades of frequency.

9.1 Parts of a Bode Plot

A Bode plot is comprised of two sub-plots. The sub-plot on top represents the magnitude of the gain over frequency. This magnitude is typically expressed in decibels. The sub-plot on the bottom represents the phase shift over frequency and can either have radians or degrees for units.

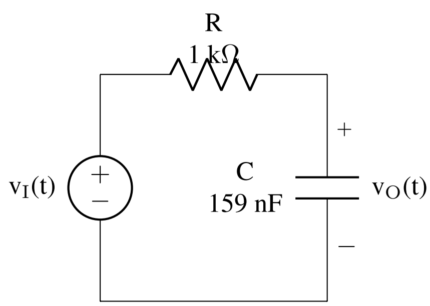
Bode Plot Example





9.2 Creating a Bode Plot

Automated tools do a far better job of generating Bode plots nowadays, however, I would like to take a slower, more labor intensive approach in this section in order to link the concept to the early chapters in this book. Let's start with a circuit



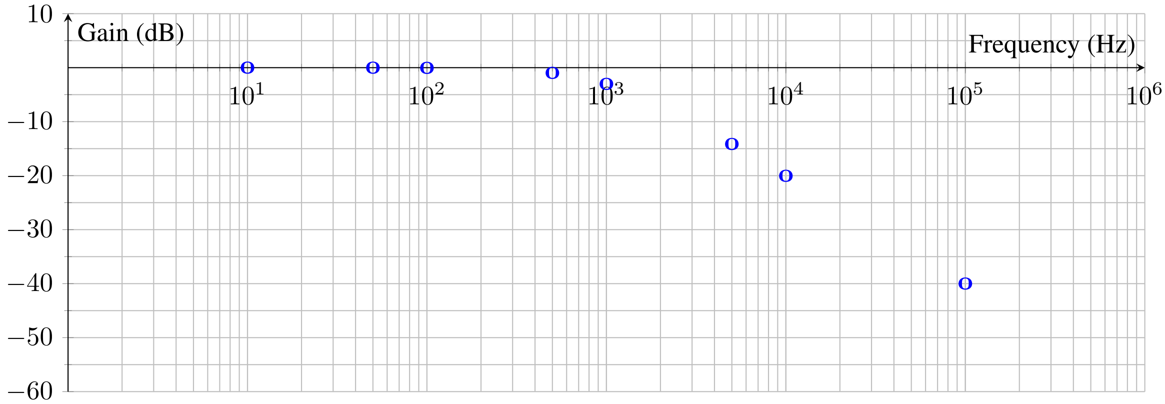
and calculate complex gain for multiple frequencies

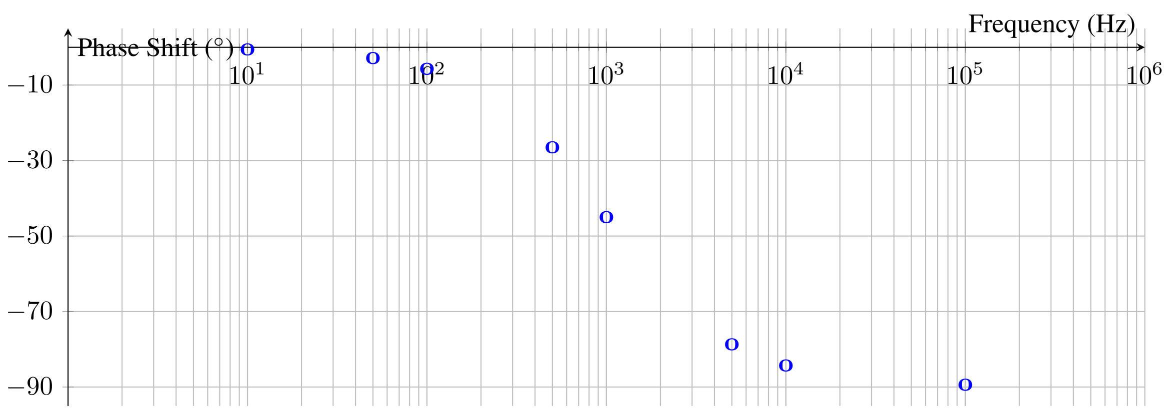
Frequency (Hz)	Gain ($^V/_V$)
10	$0.999\angle{-0.57}°$
50	$0.998\angle{-2.86}°$
100	$0.995\angle{-5.71}°$
500	$0.895\angle{-26.5}°$
1k	$0.707\angle{-45.0}°$
5k	$0.196\angle{-78.7}°$
10k	$0.099\angle{-84.3}°$
100k	$0.010\angle{-89.4}°$

The calculation of these values is covered in a previous chapter. If you need to, go re-read the sections on complex gain. These gain values are depicted by the two sub-plots of the Bode plot. Let's add two columns based on the gain column. First, we'll add the magnitude of the gain expressed in decibels. Second, we'll list the phase shift separately.

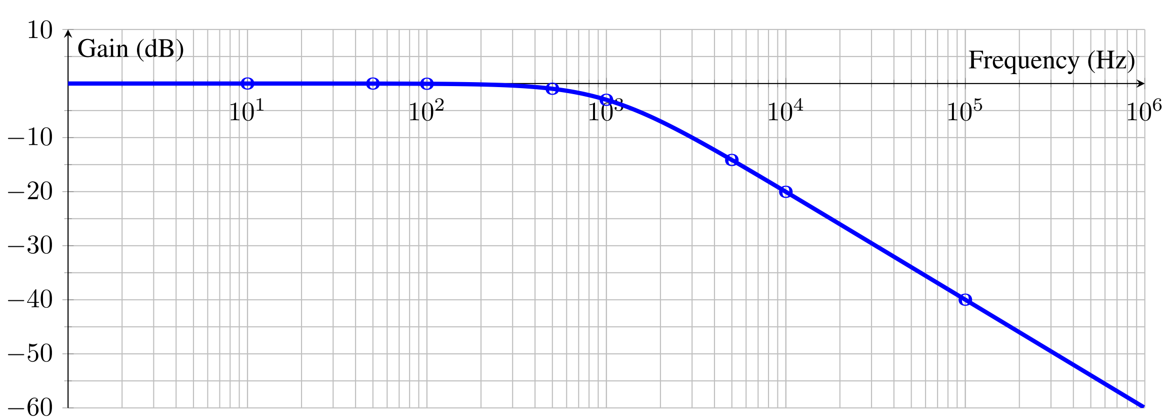
Frequency (Hz)	Gain ($^V/_V$)	Gain (dB)	Phase Shift (°)
10	$0.999\angle{-0.57°}$	-0.0004	-0.57
50	$0.998\angle{-2.86°}$	-0.0108	-2.86
100	$0.995\angle{-5.71°}$	-0.0431	-5.71
500	$0.895\angle{-26.5°}$	-0.9674	-26.5
1k	$0.707\angle{-45.0°}$	-3.0103	-45.0
5k	$0.196\angle{-78.7°}$	-14.142	-78.7
10k	$0.099\angle{-84.3°}$	-20.035	-84.3
100k	$0.010\angle{-89.4°}$	-39.992	-89.4

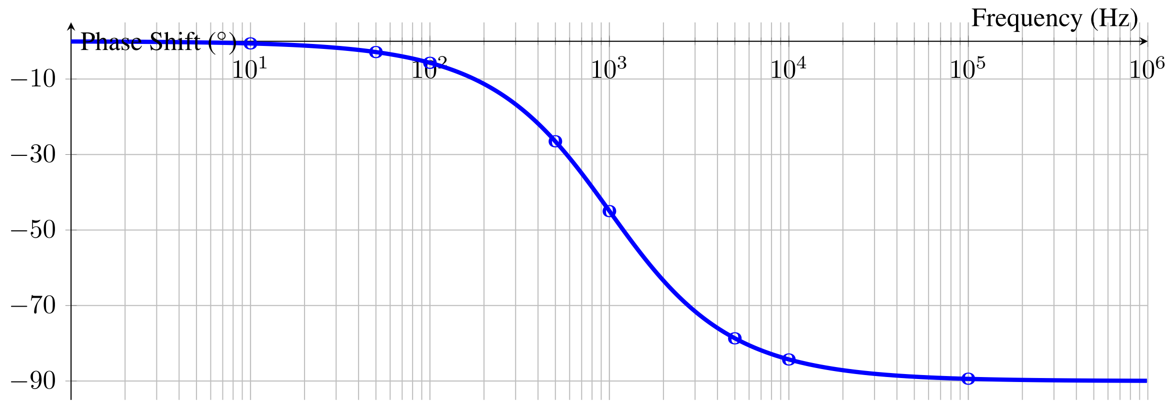
We can then locate the points from the table on a plot like this





Finally, we can connect the dots. We can visually interpolate the function that connects the dots or calculate more points where needed. Either way we are drawing a function that describes the components of gain as it varies with frequency.





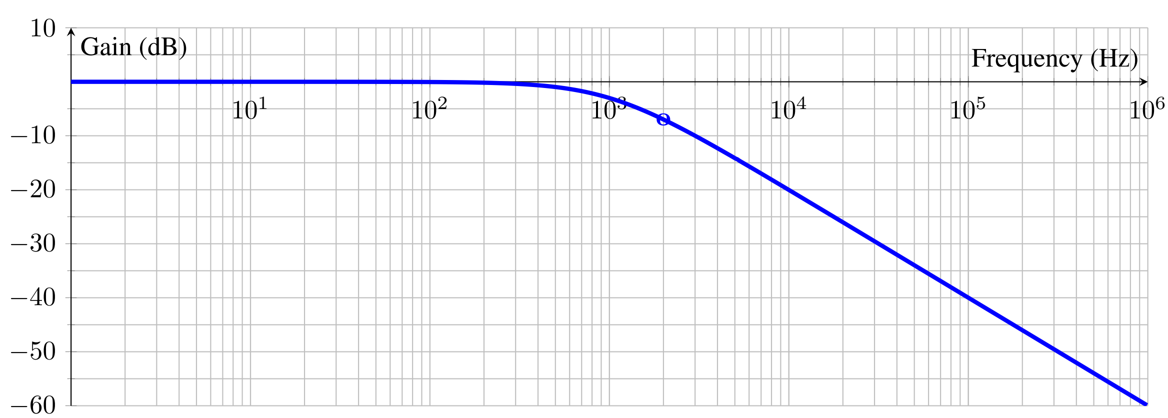
After looking at some uses for Bode plots, the next chapter will focus on finding a mathematical, rather than graphical, description of the functions depicted in the Bode plot.

9.3 Using a Bode Plot

I do not intend to show you a comprehensive list of uses for a Bode plot in this section. I want to include a few basic uses of the plots in order to connect the material to earlier chapters. Subsequent chapters will build upon this knowledge. Even in the more advanced uses I want you to recall that a Bode plot is a means of conveying how gain (in a complex form) changes as frequency varies.

9.3.1 Find the Gain

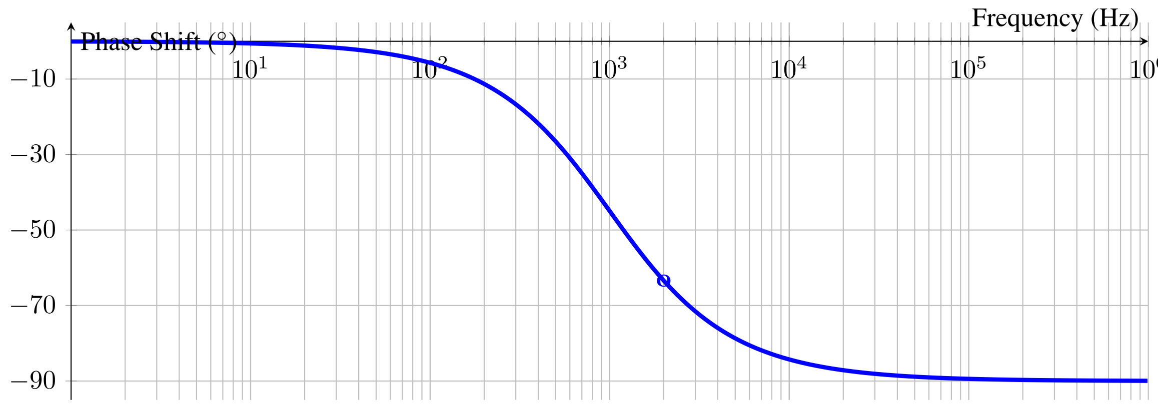
Let's continue to look at the circuit from previous sections of this chapter. This means we can also reuse the associated Bode plot we developed using the table of complex gains. Let's consider a distinct frequency that we did not calculate as part of the table, say 2 kHz. Let's answer the question: What is the voltage gain at 2 Hz? The two components of gain are treated separately by each subplot. I'll redraw the magnitude plot with 2 kHz labeled. Make sure that you can locate the frequency on the semi-log axis if it is not labeled.



After you locate the frequency along the horizontal axis, estimate the value of the function along the vertical axis. The blue dot helps us do this here but don't count on it being labeled in the future. The magnitude of the gain at 2 kHz is approximately -7 dB. I say "approximately" since anytime we read a value off of a plot we have to visually judge the value which carries with it the chance for error. To recover the complex value of the gain at 2 kHz we convert the decibel value to a scalar value

$$|A_V| = 10^{-7/20} = 0.4467\ {}^{V}\!/_{V}$$

We find the phase angle of the voltage gain using the bottom sub-plot of the Bode plot. The horizontal scales are lined up to make reading the two components easier.



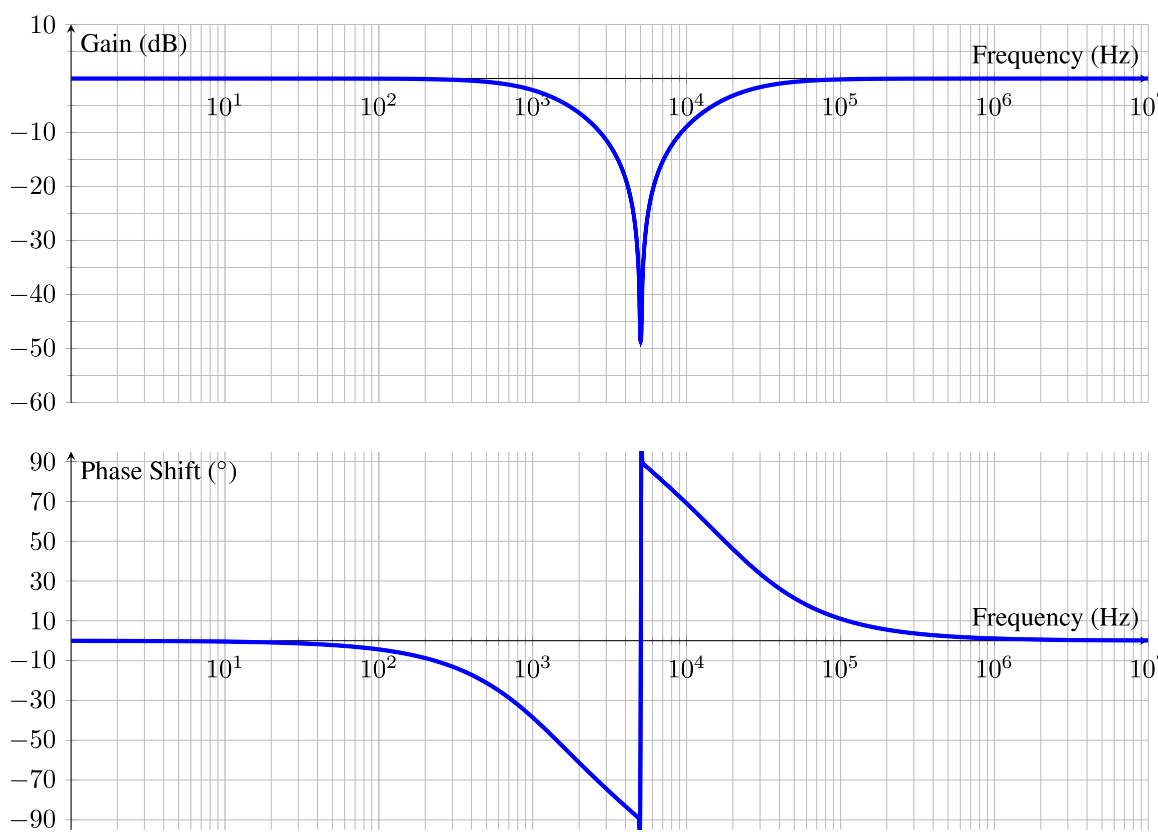
I've labeled the point again on the phase shift sub-plot. Without cheating, I approximate the phase shift at 2 kHz as -65 °. Putting the two components of gain together give us

$$A_V = 0.4467\angle{-65}° {}^V\!/\!{}_V$$

Take a moment to use the circuit shown earlier in this chapter to determine the complex gain using circuit analysis. How well did I estimate that values from the plot?

9.3.2 Determine Circuit Output at a Single Frequency

Let's consider a different circuit to get more practice reading and using Bode plots. The circuit we'll use has a Bode plot as shown here:



Notice I'm not including the schematic here. It is immaterial for our purposes here. If we know the frequency response of the circuit, as shown in the Bode plot, we can figure out the output voltage of the circuit given the input voltage. Let's try it. What is the output voltage of this circuit if the input voltage is

$$v_I(t) = 9\cos(94250t - 25°)\ V$$

This would be a good time to stop and examine the Bode plot. Find the two values we need from the plot, magnitude and phase shift at the frequency of the input. I estimate the values as -4 dB and 60° respectively. If you're very far off, look again. Did you consider the units of the frequency? $94250^{rad}/_{s}$ is equivalent to 15 kHz. If I didn't catch you, good. If I did, always check the units of the axes.

We convert the magnitude with

$$|A_V| = 10^{-4/20} = 0.631\ ^V/_V$$

and attach it to the phase-shift to get

$$A_V = 0.631\angle 60°\,^V/_V$$

Looking at $v_I(t)$ and transforming it to a phasor gives us

$$V_I = 9\angle -25°\ V$$

We can now find the output by simply multiplying the gain and input

$$V_O = A_V V_I = (0.631\angle 60°\,^V/_V)(9\angle -25°\ V) = 5.679\angle 35°\ V$$

and move use the inverse-phasor transform to move back to the time-domain

$$v_O(t) = 5.679\cos(94250t + 35°)\ V$$

If we were to build this circuit and connect a signal generator to the input that provides $v_I(t)$, we could observe $v_O(t)$ on an oscilloscope at the output.