# CHAPTER 8

# DECIBELS

## 8.1 Introduction

Our study of gain must now take a detour to describe a different way of expressing gain than we did in the previous chapter. It is more common to use a logarithmic scale to express the magnitude of a gain. This logarithmic scale is given the unit decibel after Alexander Graham Bell, inventor of the telephone.

The decibel scale is used when a comparison is made between two values. If these values span many orders of magnitude visualization will favor the larger values. We use decibels to compare values such as:

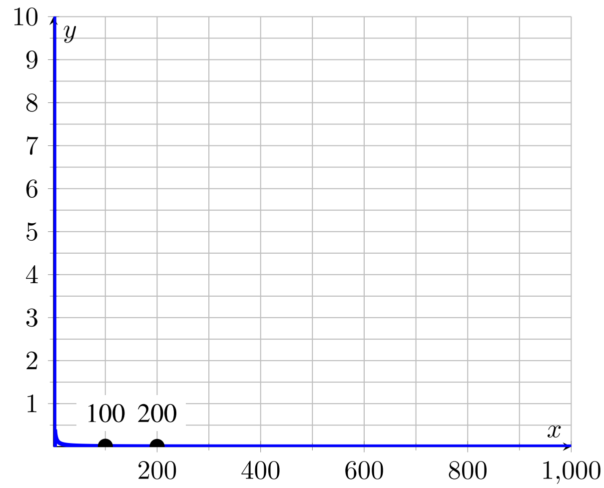
- Input and output powers (Audio or communication equipment)
- Input and output voltages/currents (Audio or communication equipment)
- Audio intensity and the threshold of hearing
- Absolute powers and a reference value (Wifi signal strength)

While logarithms are advantageous in some ways, we don't have an intuitive sense of how to use these values. To fix this we will practice working with logarithmic values and plots. Each of the next three plots represents the function

$$f(x) = \frac{1}{x}$$

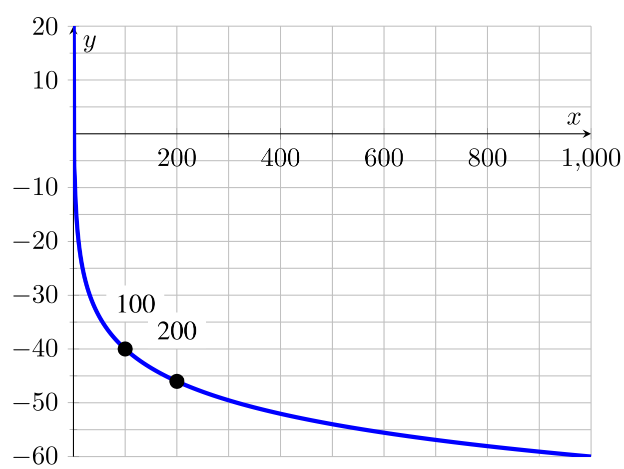
plotted between x=0.1 and x=1000. Notice each has a distinct shape depending on the type of axis and units used.

Let's examine the first plot. This plot uses a linear horizontal axis and the vertical scale is unitless (not in decibels).



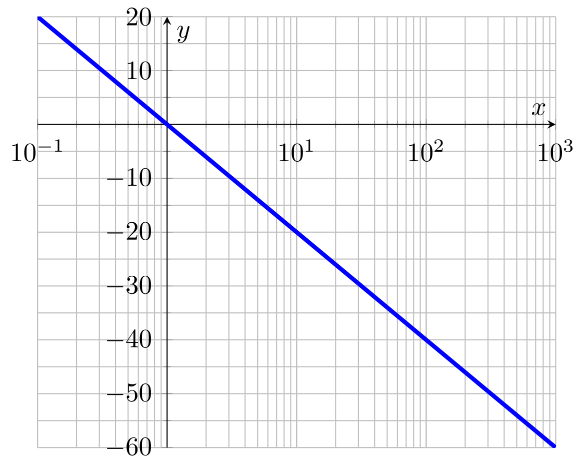

This plot is of limited use. To illustrate this, try to determine the difference between the function's value at x=100 and x=200. The two points have an indistinguishable difference in vertical position.

Here is the same function with the vertical value calculated in decibels. Let's try the same exercise. The difference between the two vertical values appears to be approximately 6 dB.




What wasn't possible with the original plot now is thanks the decibel scale. However, the improvement is not seen in all cases. Let's choose another example that remains difficult on this plot. Try to determine the difference between the function's value at x=10 and x=30. Again it is difficult to discern where to read the plot to get an accurate value at those points

The third plot addresses this problem by using a logarithmic horizontal axis. This type of plot is often called semi-logarithmic since it use a combination of linear and logarithmic scales.

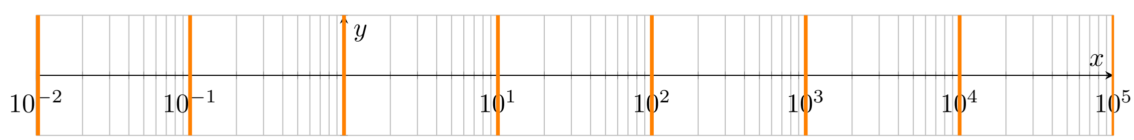



Again, try to determine the difference between the function's value at x=10 and x=30. Now we can approximate the difference as 19 dB. The semi-logarithmic plot with a decibel vertical scale has improved the utility of the figure. Of course it comes at a cost: the complexity of working with logarithms.

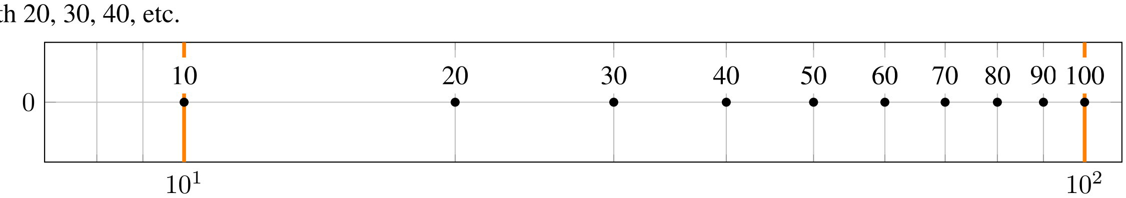
### 8.1.1 How to Use a Semi-logarithmic Axis

When we were children we learned how to count linearly. At some point we learned how to read and create plots with linear axes. Most people are more comfortable using linear axes rather than logarithmic axes. When we consider how a circuit responds to a range of frequencies over several orders of magnitude we'll use a logarithmic axis. We'll need to practice locating values along a logarithmic axis.

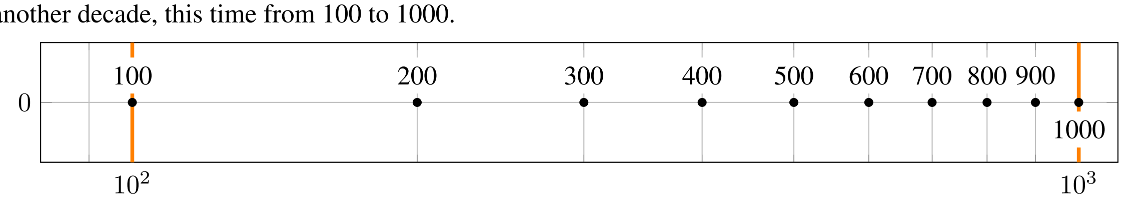
We start by locating the lines indicating the decades; that is the line indicating 0.01, 0.1, 1, 10, 100, 1000, etc. Those lines are shown in bold on the plot below.



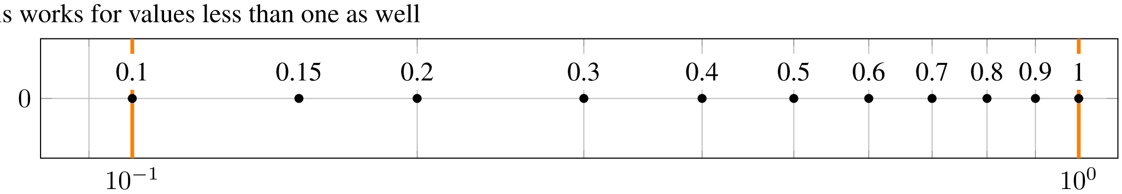

Notice that line indicating a power of ten (a decade) is followed immediately by the largest gap to the next gray line. Next, let's focus on a single decade. The decade lines are highlighted on the plot below for the values 10 and 100. In between the decade line values increment by 10. The minor grid lines are labeled on the plot starting with 20, 30, 40, etc.




This pattern is true of any decade. The minor grid has a step size equal to that of the lower decade line. Let's look at another decade, this time from 100 to 1000.

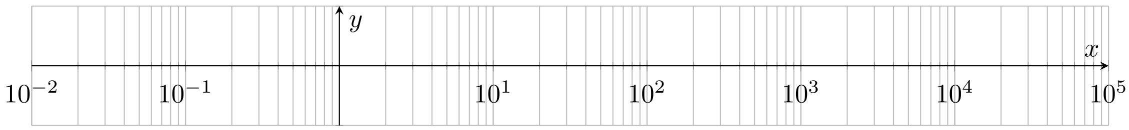



This works for values less than one as well



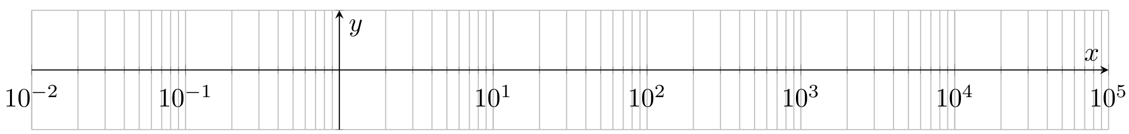
Notice that values between the minor grid lines are not linear space either. For instance, 0.15 is not half way between 0.1 and 0.2. It's labeled on the plot above closer to 0.2 rather than 0.1. Estimating values such as these takes experience. The only thing left to do is practice.

### 8.1.2 Practicing Using a Semi-logarithmic Axis



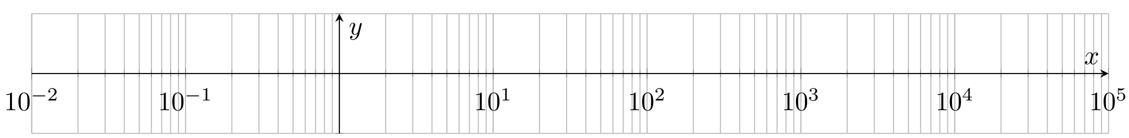

Find the following values:

1. 10
2. 100
3. 1000
4. 10000
5. 100000
6. 0.1
7. 0.01




A little trickier:

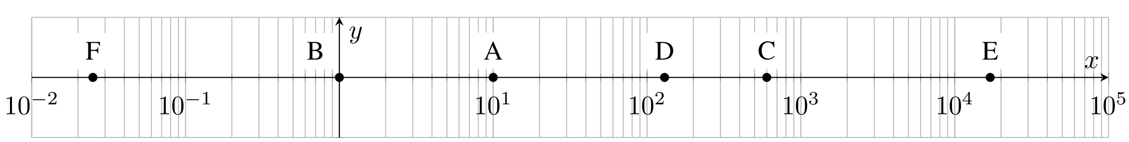
1. 20
2. 80
3. 3000
4. 40000
5. 0.6
6. 0.08




Lastly:

1. 1500
2. 45
3. 22000
4. 0.25

Determine the values labeled on the plot:




## 8.2 From Scalar Gain to Decibels

We began working with gain expressed as a scalar value sometimes called a real-valued gain. Conversion of scalar gains to the decibel scale always represents the power gain of a circuit ($A_P$).

### 8.2.1 Power Gain

We commonly assumed input and output impedances are matched when working with audio and communications equipment. Our first look at decibels will make this same assumption. The decibel gain is calculated as

$$A_{dB} = 10\,log\,(A_P) = 10\,log\left(\frac{P_{OUT}}{P_{IN}}\right)$$

At times we'll have to find the scalar gain from a decibel value. We can rearrange the equation to find scalar gain from decibel gain resulting in

$$A_p = 10^{0.1(A_{dB})}$$

Let's try a few problems to get comfortable with these equations.

**EXAMPLE 8.1**

Find the decibel gain for the following power gains:

1. 0.01 $^W/_W$
2. 0.1 $^W/_W$
3. 10 $^W/_W$
4. 100 $^W/_W$

Gains that are powers of 10, that is in the form $10^n$, have decibel equivalents that are part of a pattern. Let's calculate these values

$$A_{dB} = 10\,log\,(0.01) = -20\;dB$$
$$A_{dB} = 10\,log\,(0.1) = -10\;dB$$
$$A_{dB} = 10\,log\,(10) = 10\;dB$$
$$A_{dB} = 10\,log\,(100) = 20\;dB$$

In each case the scalar gain is in the form $10^n$, the decibel gain is equivalent to $10n$. This pattern applies to all values of $n$, not just those calculated here.

**EXAMPLE 8.2**

Find the decibel gain for the following power gains:

1. 0.5 $^W/_W$
2. 2 $^W/_W$

Gains that double the input and gains that halve the input are commonly found in problems as they will define which frequencies pass through a system and which frequencies will not. Let's calculate these values

$$A_{dB} = 10\,log\,(0.5) = -3\;dB$$
$$A_{dB} = 10\,log\,(2) = 3\;dB$$

A circuit that halves the power of the input signal has a gain of -3 dB and a circuit that doubles the power of the input signal has a gain of +3 dB. We will encounter these values a lot in future sections.

**EXAMPLE 8.3**

Find the decibel gain for the following power gains:

1. 0 $^W/_W$
2. 1 $^W/_W$

Two other gains may be of interest to us, the gain that completely attenuates an input and the gain that leaves the signal unchanged. The scalar values for these gains are 0 and 1 respectively. Let's calculate the corresponding decibel gains:

$$A_{dB} = 10\,log\,(0) = -\infty\;dB$$
$$A_{dB} = 10\,log\,(1) = 0\;dB$$

The first calculation defines one of the bounds for gain. Gain is limited to the range $[0, \infty)$. Gains that attenuate a signal are further limited to the range $[0, 1)$. Based on our calculations here gains that attenuate expressed as decibels are limited to the range $[-\infty, 0)$. We can state this a different way. **Gains that attenuate have negative decibel values.**

The gain value that does not change a signal has the scalar value 1. We sometimes call this "unity gain" because of its scalar value. The corresponding decibel gain is 0 dB.

With these bounds in minds look over all of the examples presented here. We've defined that range of values that attenuate and the value that doesn't change the signal. The rest of the values must amplify the signal. Every example used above that has a scalar gain greater than 1 also has a positive decibel value. So we can state **gains that amplify have positive decibel values.**

### 8.2.2 Voltage/Current Gain

It is not uncommon to consider circuit gains other than the power gain. However, it is important to remember that a gain in decibels is always a power gain. Let's figure out why this is by starting with a description of a circuit with input and output ports

DRAW PICTURE OF TWO-PORT SYSTEM WITH INP/OUT IMPEDANCE

Start with the definition of power gain in decibels

$$A_{dB} = 10\,log\,(A_P) = 10\,log\left(\frac{P_{OUT}}{P_{IN}}\right)$$

and define the input and output power as

$$P_{IN} = \frac{V_{IN}^2}{R_{IN}} \qquad \text{and} \qquad P_{OUT} = \frac{V_{OUT}^2}{R_{OUT}}$$

Substituting the power expressions into the power gain gives us

$$A_{dB} = 10\,log\left(\frac{P_{OUT}}{P_{IN}}\right) = 10\,log\left(\frac{V_{OUT}^2/R_{OUT}}{V_{IN}^2/R_{IN}}\right)$$

It is at this point that we can take two separate paths. We will often make the assumption that the input and output impedances are equal. Let's examine what to do when this is true and then the case where it is not.

***8.2.2.1 Matched Impedance*** In the case of matched input and output impedance the decibel gain reduces to

$$A_{dB} = 10\,log\left(\frac{P_{OUT}}{P_{IN}}\right) = 10\,log\left(\frac{V_{OUT}^2}{V_{IN}^2}\right)$$

We then rewrite the fraction as

$$A_{dB} = 10\,log\left(\frac{V_{OUT}}{V_{IN}}\right)^2$$

and using properties of logarithms (remember those?) we move the exponent to the coefficient

$$A_{dB} = 20\,log\left(\frac{V_{OUT}}{V_{IN}}\right) = 20\,log\,(A_V)$$

Notice that the logarithm is now based on the voltage gain instead of the power gain. Even though this calculation is based on $A_V$ it is important to remember how we got here. We started with the power gain expressed in decibels. This expression gives us the same type of value, a power gain expressed in decibels.

**EXAMPLE 8.4**

Find the decibel gain for the following voltage gains:

1. 0.5 $^V/_V$
2. 2 $^V/_V$

The magnitude of these are gains we used in previous examples. Here we must notice that they are voltage gains. Previously, they were power gains. Therefore, we calculated the decibel equivalents as:

$$A_{dB} = 20\ log\,(0.5) = -6\ dB$$
$$A_{dB} = 20\ log\,(2) = 6\ dB$$

A circuit that halves the power of the input signal has a gain of -3 dB but a circuit that halves the voltage has a gain of -6 dB. Similarly, a circuit that doubles the power of the input signal has a gain of +3 dB but a circuit that doubles the voltage has a gain of 6 dB.

**EXAMPLE 8.5**

What voltage gain corresponds to the following power gains:

1. 0.5 $^{W}/_{W}$
2. 2 $^{W}/_{W}$

Let's start with the half-power gain, 0.5 $^{W}/_{W}$. We know from previous calculation that it is equivalent to -3 dB. Using the decibel value in the expression for decibels based on voltage gain looks like this

$$-3\ dB = 20\ log\,(A_V)$$

Solving for $A_V$ give us

$$A_V = 10^{-\,^{3}/_{20}} = \frac{1}{\sqrt{2}} \approx 0.707$$

Moving to the double-power gain, 2 $^{W}/_{W}$. We know from previous calculation that it is equivalent to +3 dB. Using the decibel value in the expression for decibels based on voltage gain looks like this

$$+3\ dB = 20\ log\,(A_V)$$

Solving for $A_V$ give us

$$A_V = 10^{^{3}/_{20}} = \sqrt{2} \approx 1.414$$

These values should be familiar to you my now as the relate the magnitude of a phasor with angle $\pm 45°$ to its real component.

***8.2.2.2 Unmatched Impedance*** Now let's take a step back to before we made the assumption that the input and output impedances were matched. We were working on relating power gain in decibels to the input and output voltages. We left off with this expression

$$A_{dB} = 10\ \log\left(\frac{P_{OUT}}{P_{IN}}\right) = 10\ \log\left(\frac{V_{OUT}^2/R_{OUT}}{V_{IN}^2/R_{IN}}\right)$$

In cases that we cannot assume $R_{IN}$ and $R_{OUT}$ are the same we rely on more trigonometric identities to pull the resistances out of the gain ratio

$$A_{dB} = 10\ \log\left(\frac{V_{OUT}^2}{V_{IN}^2}\frac{R_{IN}}{R_{OUT}}\right) = 10\ \log\left(\frac{V_{OUT}^2}{V_{IN}^2}\right) + 10\ \log\left(\frac{R_{IN}}{R_{OUT}}\right)$$

Which we simplify in the same manner as in the matched-impedance case

$$A_{dB} = 10\ \log\left(\frac{V_{OUT}}{V_{IN}}\right)^2 + 10\ \log\left(\frac{R_{IN}}{R_{OUT}}\right) = 20\ \log\left(\frac{V_{OUT}}{V_{IN}}\right) + 10\ \log\left(\frac{R_{IN}}{R_{OUT}}\right)$$

| | Ratio | Power Ratio(dB) | Voltage Ratio(dB) |
|---|---|---|---|
| Attenuation | 0 | -∞ | -∞ |
| | 0.1 | -10 | -20 |
| | 0.5 | -3 | -6 |
| Unity Gain | 1 | 0 | 0 |
| Amplification | 2 | 3 | 6 |
| | 10 | 10 | 20 |
| | 100 | 20 | 40 |

**Table 8.1** Notable decibel values

## 8.3 From Complex Gain to Decibels

## 8.4 Cascaded System Gain

## 8.5 Absolute Power in Decibels

Decibels are used to measure ratios other than gain. Decibels are a convenient unit in many circuits with very large and very small magnitudes of power. Even though we are using decibels to measure absolute power it is still a comparison of two values. Decibels always compare two values. In the case of absolute power we compare the power level to a reference level. Common values for this reference include 1 W, 1 mW, and 1 fW. Let's look at how we move between absolute and decibel values of power.

### 8.5.1 Basic Definition

The power in Watts can be converted to the corresponding decibel value using the formulas in the left column. The different rows correspond to separate reference levels. The power as a decibel value can be converted back to an absolute value using the formulas in the right column.

$$\begin{aligned} P(\text{dBW}) &= 10 \log \frac{P}{1\ W} & P &= (1\ W)\left[10^{\frac{P(\text{dBW})}{10}}\right] \\ P(\text{dBm}) &= 10 \log \frac{P}{1\ mW} & P &= (1\ mW)\left[10^{\frac{P(\text{dBm})}{10}}\right] \\ P(\text{dBf}) &= 10 \log \frac{P}{1\ fW} & P &= (1\ fW)\left[10^{\frac{P(\text{dBf})}{10}}\right] \end{aligned}$$

Time for some practice

**EXAMPLE 8.6**

Given a voltage V in volts:

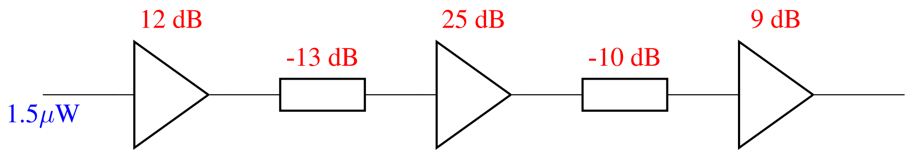
$$V(\text{dBV}) = 20 \log \frac{V}{1\ V}$$

$$V(\text{dBmV}) = 20 \log \frac{V}{1\ mV}$$

$$V(\text{dB}\mu\text{V}) = 20 \log \frac{V}{1\ \mu V}$$

### 8.5.2 Quick Conversion Between Power Scales

## 8.6 Cascaded System Gain




What is the power at the output of the cascaded amplifier connected by the transmission lines? If 14dBm is measured at the output, what is the input?