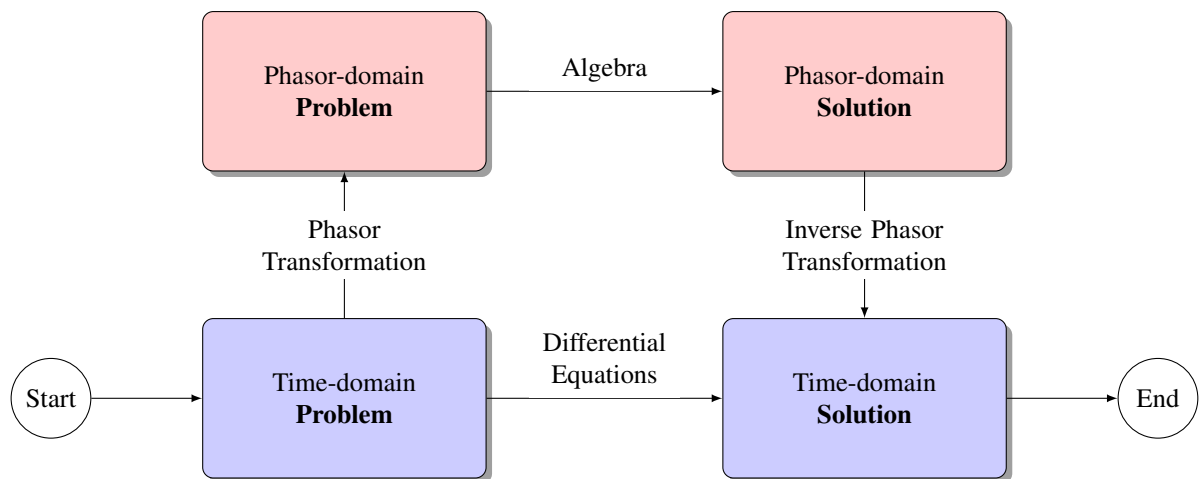


## CHAPTER 4

---

# CIRCUIT ANALYSIS WITH PHASORS

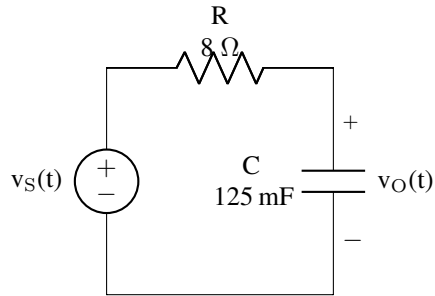
---



### 4.1 Voltage and Current Dividers

#### 4.1.1 Simple Dividers

##### ■ EXAMPLE 4.1



Find  $v_O(t)$  given  $v_S(t) = 12 \cos(4t + 40^\circ) V$

All three components need to be moved to the phasor domain. There is a single frequency for this circuit and it is the coefficient of  $t$  in  $v_S(t)$ . In this case  $\omega = 4 \text{ rad/s}$

$$V_S = 12 \angle 40^\circ V$$

$$Z_R = R = 8 \Omega$$

$$Z_C = \frac{1}{j\omega C} = -j2 \Omega$$

The circuit analysis for these early examples will be kept simple. Here the circuit forms a voltage divider. We can use the voltage divider formula with the impedances.

$$V_O = V_S \left[ \frac{Z_C}{Z_C + Z_R} \right] = (12 \angle 40^\circ V) \left[ \frac{-j2 \Omega}{8 - j2 \Omega} \right] = 2.91 \angle -35.96^\circ V$$

You should then move the phasor result back to the time-domain

$$v_O(t) = 2.91 \cos(4t - 35.96^\circ) V$$

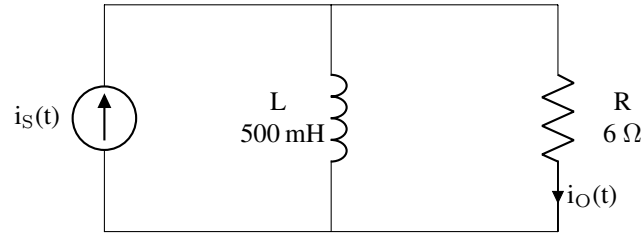
This can be done in MATLAB using the script shown below.

```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 R=8;%Ohm
7 C=125e-3;%F
8 w=4;%rad/s
9
10 %Move to phasor-domain
11 Vs=12*exp(j*40*(pi/180));
12 Zr=R;
13 Zc=1/(j*w*C);
14
15 %Perform circuit analysis
16 Vo=Vs*(Zc/(Zc+Zr));
17
18 %Move back to the time-domain
19 fprintf('v_O(t)=%.2fcos(%.2ft%+.2f deg) V\n',abs(Vo),w,angle(Vo)*(180/pi))

```

#### EXAMPLE 4.2



Find  $i_O(t)$  given  $i_S(t) = 3 \cos(8t - 25^\circ) A$

All three components need to be moved to the phasor domain. There is a single frequency for this circuit and it is the coefficient of  $t$  in  $i_S(t)$ . In this case  $\omega = 8 \text{ rad/s}$

$$I_S = 3 \angle -25^\circ V$$

$$Z_R = R = 6 \Omega$$

$$Z_L = j\omega L = j4 \Omega$$

Here the circuit forms a current divider. We can use the current divider formula with the impedances.

$$I_O = I_S \left[ \frac{Z_L}{Z_L + Z_R} \right] = (3 \angle -25^\circ A) \left[ \frac{j4 \Omega}{6 + j4 \Omega} \right] = 1.66 \angle 31.31^\circ A$$

You should then move the phasor result back to the time-domain

$$i_O(t) = 1.66 \cos(8t + 31.31^\circ) A$$

This can be done in MATLAB using the script shown below.

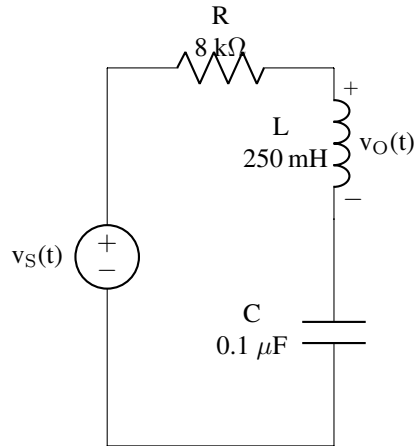
```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 R=6;%Ohm
7 L=500e-3;%H
8 w=8;%rad/s
9
10 %Move to phasor-domain
11 Is=3*exp(j*-25*(pi/180));
12 Zr=R;
13 Zl=j*w*L;
14
15 %Perform circuit analysis
16 Io=Is*(Zl/(Zl+Zr));
17
18 %Move back to the time-domain
19 fprintf('i_O(t)=%.2f*cos(%.2ft%+.2f deg) A\n',abs(Io),w,angle(Io)*(180/pi))

```

#### 4.1.2 Dividers with Three or More Elements

##### ■ EXAMPLE 4.3



Find  $v_O(t)$  given  $v_S(t) = 10 \cos(2000t + 40^\circ)$  mV

All four components need to be moved to the phasor domain. There is a single frequency for this circuit and it is the coefficient of  $t$  in  $v_S(t)$ . In this case  $\omega = 2000\text{ rad/s}$

$$V_S = 10 \angle 40^\circ \text{ mV}$$

$$Z_R = R = 8 \text{ k}\Omega$$

$$Z_L = j\omega L = j500 \Omega$$

$$Z_C = \frac{1}{j\omega C} = -j5 \text{ k}\Omega$$

Here the circuit forms a voltage divider.  $R$  and  $C$  in this circuit are in series and can be therefore be combined into a single component with impedance  $Z_R + Z_C$ . They are in series since they are both in series with  $L$  and carry the same current. We can use the voltage divider formula with the impedances.

$$V_O = V_S \left[ \frac{Z_L}{Z_L + Z_R + Z_C} \right] = (10 \angle 40^\circ \text{ mV}) \left[ \frac{j500 \Omega}{8\text{ k} - 4.5\text{ k} \Omega} \right] = 544.7 \angle 159.4^\circ \mu\text{V}$$

You should then move the phasor result back to the time-domain

$$v_O(t) = 544.7 \cos(2000t + 159.4^\circ) \mu\text{V}$$

This can be done in MATLAB using the script shown below.

```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 R=8e3;%Ohm
7 C=0.1e-6;%F
8 L=250e-3;%H
9 w=2000;%rad/s
10
11 %Move to phasor-domain
12 Vs=10e-3*exp(j*40*(pi/180));
13 Zr=R;
14 Zc=1/(j*w*C);
15 Zl=j*w*L;
16
17 %Perform circuit analysis
18 Vo=Vs*(Zl/(Zl+Zc+Zr));

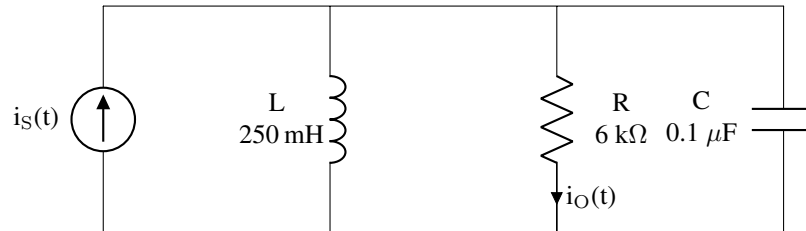
```

```

19
20 %Move back to the time-domain
21 fprintf('v_O(t)=%.2f*cos(%.2ft%+.2f deg) uV\n', 1e6*abs(Vo), w, angle(Vo)*(180/
    pi))

```

#### EXAMPLE 4.4



Find  $i_O(t)$  given  $i_S(t) = 5 \cos(4000t - 65^\circ)$  mA

All four components need to be moved to the phasor domain. There is a single frequency for this circuit and it is the coefficient of  $t$  in  $i_S(t)$ . In this case  $\omega = 4000 \text{ rad/s}$

$$I_S = 5 \angle -65^\circ \text{ mA}$$

$$Z_R = R = 6 \text{ k}\Omega$$

$$Z_L = j\omega L = j4 \Omega$$

$$Z_C = \frac{1}{j\omega C} = -j5 \text{ k}\Omega$$

Here the circuit forms a current divider. In this circuit  $L$  and  $C$  are in parallel as both are connected to the same two nodes. You need to combine  $L$  and  $C$  prior to applying the current divider.

$$Z_P = Z_C || Z_L = j1.67 \text{ k}\Omega$$

We can use the current divider formula with the impedances.

$$I_O = I_S \left[ \frac{Z_P}{Z_P + Z_R} \right] = (5 \angle -65^\circ \text{ mA}) \left[ \frac{j1.67 \text{ k}\Omega}{6 \text{ k} + j1.67 \text{ k}\Omega} \right] = 1.34 \angle 9.48^\circ \text{ mA}$$

You should then move the phasor result back to the time-domain

$$i_O(t) = 1.34 \cos(4000t + 9.48^\circ) \text{ mA}$$

This can be done in MATLAB using the script shown below.

```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
    notation
5
6 R=6e3;%Ohm
7 L=250e-3;%H
8 C=0.1e-6;%F
9 w=4e3;%rad/s
10
11 %Move to phasor-domain
12 Is=5e-3*exp(j*-65*(pi/180));
13 Zr=R;
14 Zc=1/(j*w*C);

```

```

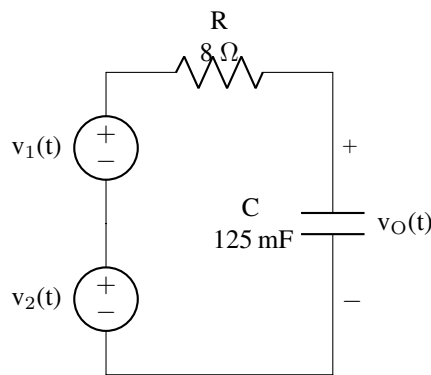
15 Z1=j*w*L;
16
17 %Perform circuit analysis
18 Zp=(Z1*Zc)/(Z1+Zc);
19 Io=Is*(Zp/(Zp+Zr));
20
21 %Move back to the time-domain
22 fprintf('i_0(t)=%.2f*cos(%.2ft%+.2f deg) mA\n',1e3*abs(Io),w,angle(Io)*(180/
    pi))

```

### 4.1.3 Dividers with Multiple Supplies

The next group of examples is leading to an important part of the course that will be covered in later chapters. Signals can be broken down into multiple sinusoidal signals. These signals can be analyzed as if they were generated using many voltage supplies in series or many current supplies in parallel.

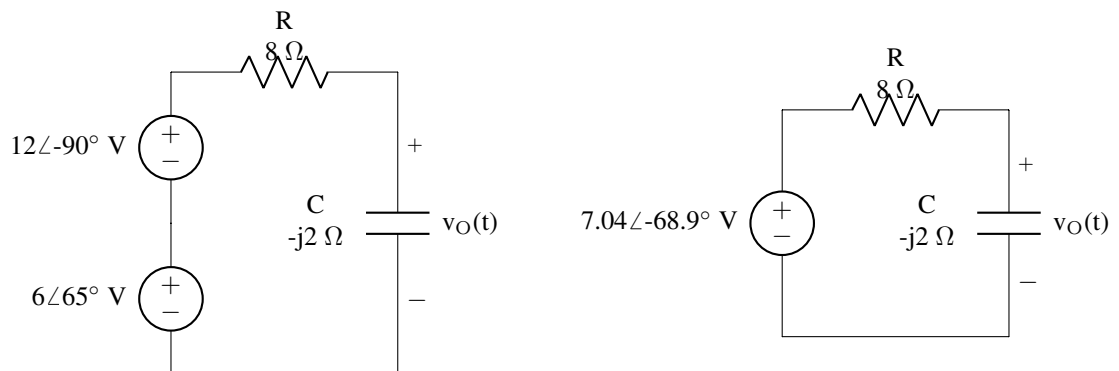
#### EXAMPLE 4.5



Find  $v_O(t)$  given  $v_1(t) = 12 \cos(4t - 90^\circ) \text{ V}$  and  $v_2(t) = 6 \cos(4t + 65^\circ) \text{ V}$

The voltage supplies are connected in series so the values can be added together and the sources combined. We should consider in which domain to add the voltages. We can always add the voltages in the time-domain however, in this case there is a better option. Since both sources are operating at the same frequency,  $4 \text{ rad/s}$  in this case, the voltages can be added as phasors. This will reduce the amount of analysis we will have to do. We will treat the case where we cannot do this, when the frequencies are different, in an example later on.

First we move the problem to the phasor domain as shown below on the left. Next, we combine the voltage sources resulting in the circuit shown below on the right.



The circuit analysis is now a simple application of the voltage divider:

$$V_O = V_S \left[ \frac{Z_C}{Z_C + Z_R} \right] = (7.04 \angle -68.9^\circ \text{ V}) \left[ \frac{-j2}{8 - j2} \right] = 1.71 \angle -144.9^\circ \text{ V}$$

Finally, we move the solution back to the time-domain

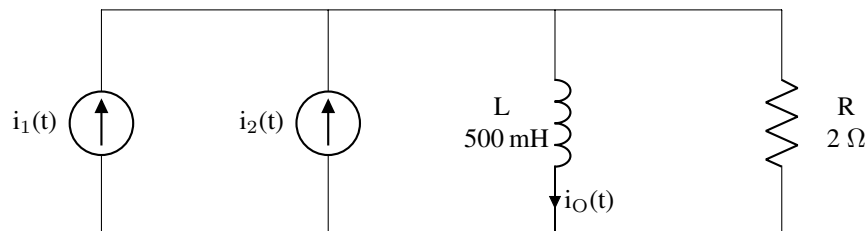
$$1.71 \cos(4t - 144.9^\circ) \text{ V}$$

```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 R=8;%Ohm
7 C=125e-3;%F
8 w=4;%rad/s
9
10 %Move to phasor-domain
11 V1=12*exp(j*-90*(pi/180));
12 V2=6*exp(j*65*(pi/180));
13 Zr=R;
14 Zc=1/(j*w*C);
15
16 %Perform circuit analysis
17 Vs=V1+V2;
18 Vo=Vs*(Zc/(Zc+Zr));
19
20 %Move back to the time-domain
21 fprintf('v_o(t)=%.2f*cos(%.2ft+%.2f deg) V\n',abs(Vo),w,angle(Vo)*(180/pi))

```

#### EXAMPLE 4.6

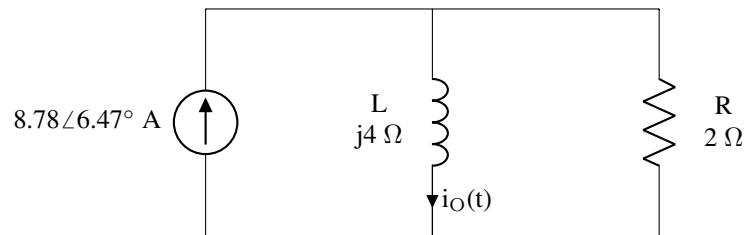


Find  $i_O(t)$  given  $i_1(t) = 5 \cos(8t + 125^\circ) \text{ A}$  and  $i_2(t) = 12 \cos(8t - 15^\circ) \text{ A}$

This problem is similar to the last given the current supplies are connected in parallel and are operating at the same frequency. Let's move to the phasor domain

$$I_1 = 5 \angle 125^\circ \text{ A} \quad I_2 = 12 \angle -15^\circ \text{ A} \quad Z_L = j4 \Omega \quad Z_R = 2 \Omega$$

and then combine the supplies.



The circuit analysis is now a simple application of the current divider:

$$I_O = I_S \left[ \frac{Z_R}{Z_L + Z_R} \right] = (8.78 \angle 6.47^\circ \text{ A}) \left[ \frac{2}{2 + j4} \right] = 3.93 \angle -57.0^\circ \text{ A}$$

Finally, we move the solution back to the time-domain

$$3.93 \cos(8t - 57.0^\circ) \text{ V}$$

```

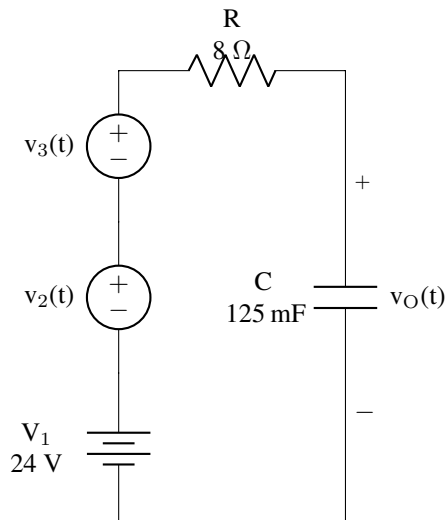
1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 R=2;%Ohm
7 L=500e-3;%H
8 w=8;%rad/s
9
10 %Move to phasor-domain
11 I1=5*exp(j*125*(pi/180));
12 I2=12*exp(j*-15*(pi/180));
13 Zr=R;
14 Zl=j*w*L;
15
16 %Perform circuit analysis
17 Is=I1+I2;
18 Io=Is*(Zr/(Zl+Zr));
19
20 %Move back to the time-domain
21 fprintf('i_O(t)=%.2fcos(%.2ft%+.2f deg) A\n',abs(Io),w,angle(Io)*(180/pi))

```

#### EXAMPLE 4.7

It's time for us to consider how to analyze a circuit with sources operating at different frequencies. It's been said, mostly by me, that superposition is a horrible way to analyze a circuit but sometime it is the only way. We have arrived at a problem that requires us to use superposition. Superposition requires the repeated analysis of a circuit which can be tedious. When we have to use superposition, we can also use MATLAB to relieve us of the tedium of repeating the analysis.

The example shown here is simple enough we do not need to resort to MATLAB, however, the code is included as always.



Given:

$$v_2(t) = 8 \cos(4t - 120^\circ) \text{ V}$$



$$v_3(t) = 10 \sin(8t + 55^\circ) V$$

Find  $v_O(t)$

Again, we notice that the voltage supplies are connected in series, however, this time they are operating at different frequencies. This forces us to only combine voltages in the time-domain only. To use superposition we break the problem into to three “sub-problems”, one for each independent supply.

### Sub-problem #1

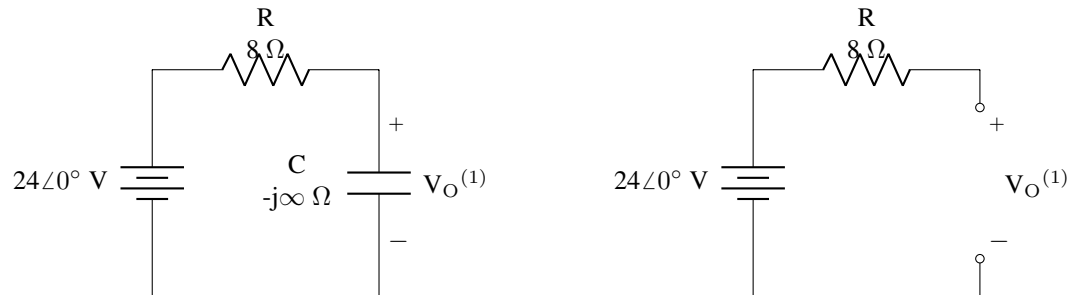
The first supply we will consider is the DC source. By definition, a DC source does not vary as a function of time. So how do we treat this source in the context of phasor analysis? Each source is assigned a phasor defined with an amplitude, angular frequency, and phase angle. So we choose the values to ensure a constant 24 V for this source.

$$24 V = A \cos(\omega t + \theta) V$$

If we choose A to be 24 V we simply have to reduce the cosine to be equivalent to 1 no matter the value of  $t$ . Selecting  $\omega$  as  $0 \text{ rad/s}$  and  $\theta$  as  $0^\circ$  accomplishes this.

$$24 V = 24 \cos(0t + 0^\circ) V$$

Now the DC source can be represented as a phasor with a value of  $24\angle 0^\circ$  and an angular frequency of  $0 \text{ rad/s}$ . We can now replace the other independent supplies with their ideal impedance and move the problem to the phasor-domain as shown below on the left.



The capacitor appears as an open as shown in the circuit above on the right. The only circuit analysis for this problem is application of the voltage divider.

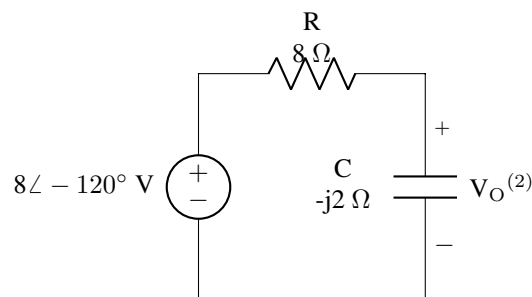
$$V_O^{(1)} = (24\angle 0^\circ) \left[ \frac{-j\infty}{8 - j\infty} \right] = 24\angle 0^\circ V$$

Moving back to the time-domain give us

$$v_o^{(1)}(t) = 24 \cos(0t + 0^\circ) V = 24 V$$

### Sub-problem #2

We now consider how the  $v_2$  supply affects the output. Replacing the other independent supplies and moving to the phasor domain yields



applying the voltage divider again gives us

$$V_O^{(2)} = (8\angle -120^\circ) \left[ \frac{-j2}{8 - j2} \right] = 1.94\angle 164.0^\circ V$$

which is

$$v_o^{(2)}(t) = 1.94 \cos(4t + 164.0^\circ) \text{ V}$$

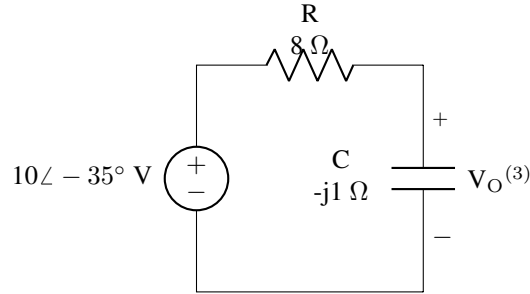
in the time-domain.

### Sub-problem #3

Finally we consider how the  $v_3$  supply affects the output. Notice that this voltage source is a sine and needs to be converted to an equivalent cosine

$$v_3(t) = 10 \sin(8t + 55^\circ) \text{ V} = 10 \cos(8t - 35^\circ) \text{ V}$$

Replacing the other independent supplies and moving to the phasor domain yields



applying the voltage divider again gives us

$$V_O^{(3)} = (10\angle -35^\circ) \left[ \frac{-j1}{8-j1} \right] = 1.24\angle -117.9^\circ \text{ V}$$

which is

$$v_o^{(3)}(t) = 1.24 \cos(8t - 117.9^\circ) \text{ V}$$

in the time-domain

### Summing the Results

The results of each sub-problem are phasors at different frequencies. Therefore we can only sum those results in the time-domain

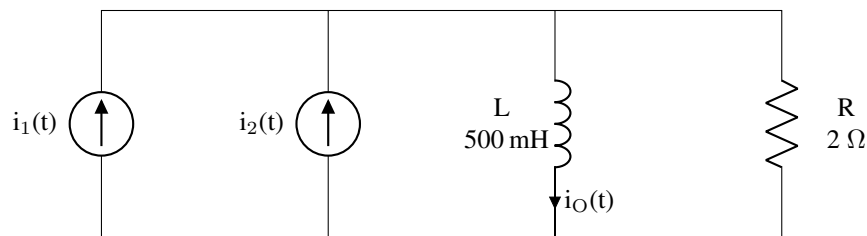
$$v_o(t) = v_o^{(1)}(t) + v_o^{(2)}(t) + v_o^{(3)}(t)$$

or

$$v_o(t) = 24 + 1.94 \cos(4t + 164.0^\circ) + 1.24 \cos(8t - 117.9^\circ) \text{ V}$$

### EXAMPLE 4.8

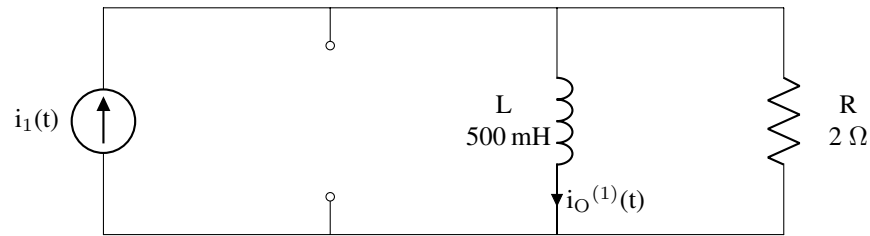
The example shown here is simple enough we do not need to resort to MATLAB, however, the code is included as always.



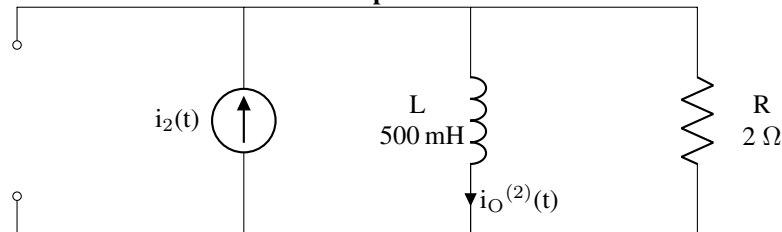
Find  $i_O(t)$  given  $i_1(t) = 4 \cos(8t + 25^\circ) \text{ A}$  and  $i_2(t) = 15 \cos(3t - 20^\circ) \text{ A}$ .

Again, we notice that the current supplies are connected in parallel, however, this time they are operating at different frequencies. This forces us to only combine currents in the time-domain only. To use superposition we break the problem into two “sub-problems”, one for each independent supply.

### Sub-problem #1



Sub-problem #2



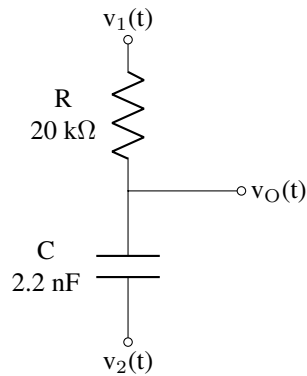
```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 L=500e-3;%H
7 R=2;%Ohm
8
9 %Move to phasor-domain
10 %sub-problem #1
11 w1=8;%rad/s
12 Z1=j*w1*L;
13 Zr=2;
14 I1=4*exp(j*25*(pi/180));
15
16 %solve sub-problem #1
17 Io1=I1*(Zr/(Zr+Z1));
18
19 %sub-problem #2
20 w2=3;%rad/s
21 Z1=j*w2*L;
22 Zr=2;
23 I2=15*exp(j*-20*(pi/180));
24
25 %solve sub-problem #2
26 Io2=I2*(Zr/(Zr+Z1));
27
28 %move back to time-domain
29 fprintf('i_o1(t)=%.2fcos(%.2ft%+.2f deg) A\n',abs(Io1),w1,angle(Io1)*(180/
   pi));
30 fprintf('i_o2(t)=%.2fcos(%.2ft%+.2f deg) A\n',abs(Io2),w2,angle(Io2)*(180/
   pi));
31
32 %Finish superposition
33 fprintf('i_o(t)=%.2fcos(%.2ft%+.2f deg)%+.2fcos(%.2ft%+.2f deg) A\n',abs(
   Io1),w1,angle(Io1)*(180/pi),abs(Io2),w2,angle(Io2)*(180/pi));

```

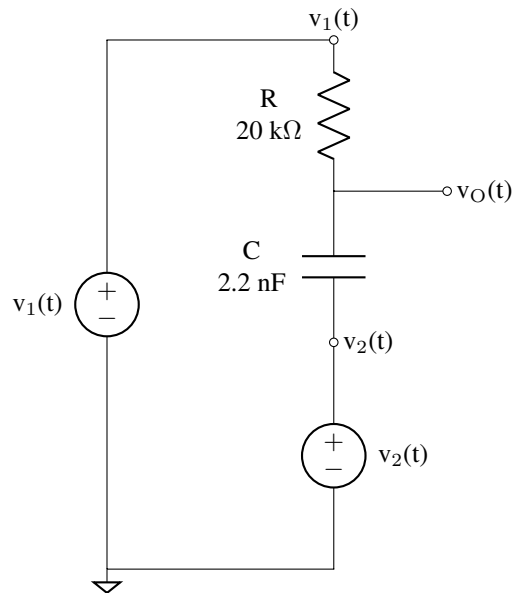
## 4.1.4 Dividers in Split Supply Circuits

## EXAMPLE 4.9



$$v_1(t) = 10 \cos(10000t + 40^\circ) \text{ mV}$$

$$v_2(t) = -10 \cos(8000t - 20^\circ) \text{ mV}$$



```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 C=2.2e-9;%F
7 R=20e3;%Ohm
8
9 %Move to phasor-domain
10 %sub-problem #1
11 w1=10000;%rad/s
12 Zc=1/(j*w1*C);
13 Zr=R;
14 V1=10e-3*exp(j*40*(pi/180));

```

```

15
16 %solve sub-problem #1
17 Vo1=V1*(Zc/(Zr+Zc));
18
19 %sub-problem #2
20 w2=8000;%rad/s
21 Zc=1/(j*w2*C);
22 Zr=R;
23 V2=-10e-3*exp(j*-20*(pi/180));
24
25 %solve sub-problem #2
26 Vo2=V2*(Zr/(Zr+Zc));
27
28 %move back to time-domain
29 fprintf('V_o1(t)=%.2fcos(%.2ft%+.2f deg) mV\n',abs(Vo1)*1000,w1,angle(Vo1)
      *(180/pi));
30 fprintf('V_o2(t)=%.2fcos(%.2ft%+.2f deg) mV\n',abs(Vo2)*1000,w2,angle(Vo2)
      *(180/pi));
31
32 %Finish superposition
33 fprintf('V_o(t)=%.2fcos(%.2ft%+.2f deg)+%.2fcos(%.2ft%+.2f deg) mV\n',abs(
      Vo1)*1000,w1,angle(Vo1)*(180/pi),abs(Vo2)*1000,w2,angle(Vo2)*(180/pi));

```

#### 4.1.5 Designing Dividers

##### EXAMPLE 4.10

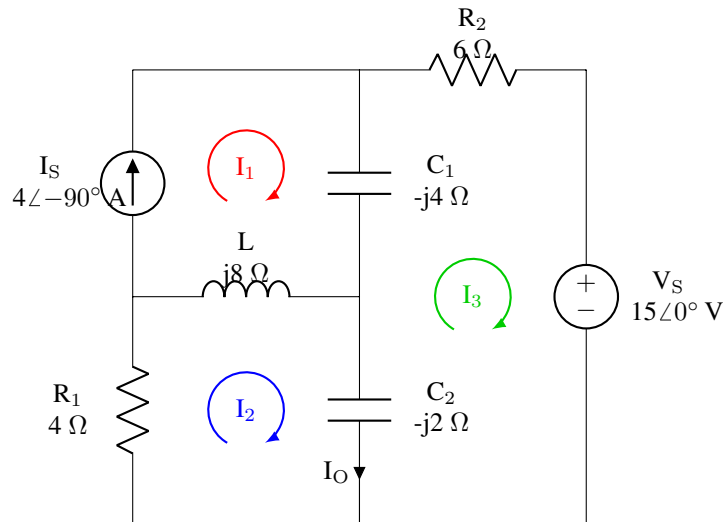
You are tasked with designing a circuit to monitor a 12 V lead-acid battery as it discharges and the voltage drops to 0 V. The voltage across the battery will be measured by an Atmel Atmega1284P microcontroller via the analog to digital converter. Consider all components as non-ideal in your design.

If 5% resistors are used what is the maximum error you expect in the output?

#### 4.1.6 Sensitivity in Dividers

#### 4.2 Mesh Analysis

##### EXAMPLE 4.11



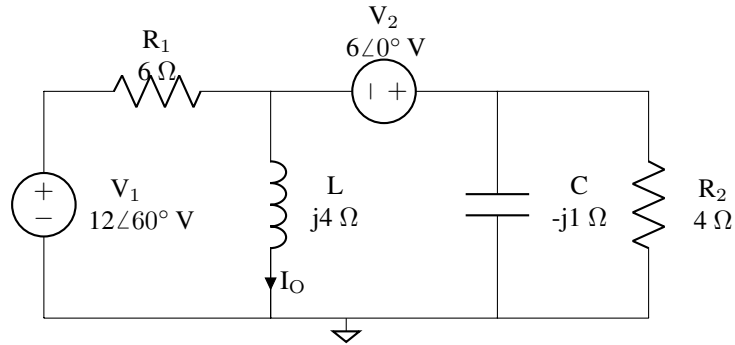
```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 %This problem is already in the phasor-domain
7 Is=4*exp(j*-90*(pi/180));
8 Vs=15*exp(j*0*(pi/180));
9
10 Zr1=4;%Ohm
11 Zr2=6;%Ohm
12 Zl=j*8;%Ohm
13 Zc1=-j*4;%Ohm
14 Zc2=-j*2;%Ohm
15
16 %setup symbolic variables for mesh analysis
17 syms I1 I2 I3
18 eqn(1)=I1==Is;
19 eqn(2)=-Zr1*I2-Zl*(I2-I1)-Zc2*(I2-I3)==0;
20 eqn(3)=-Zc2*(I3-I2)-Zc1*(I3-I1)-Zr2*I3-Vs==0;
21 sol=solve(eqn,I1,I2,I3);
22
23 %Phasor solution will be printed in rectangular and polar form
24 Io=eval(vpa(sol.I2-sol.I3,3))
25 fprintf('%0.2f at an angle of %0.2f degrees\n',abs(Io),angle(Io)*(180/pi));

```

### 4.3 Nodal Analysis

#### EXAMPLE 4.12



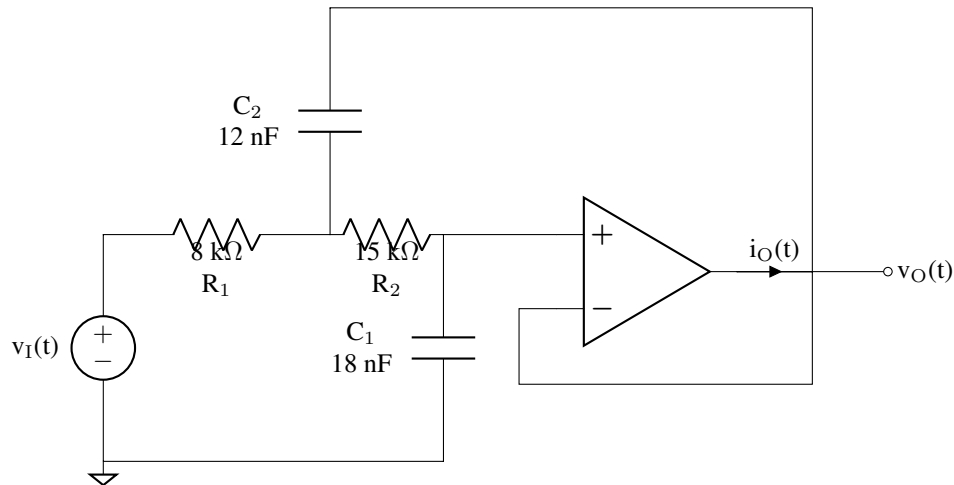
```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 %This problem is already in the phasor-domain
7 V1=12*exp(j*60*(pi/180));
8 V2=6*exp(j*0*(pi/180));
9
10 Zr1=6;%Ohm
11 Zr2=4;%Ohm
12 Zl=j*4;%Ohm
13 Zc=-j*1;%Ohm
14
15 %setup symbolic variables for mesh analysis
16 syms Va Vb Vc
17 eqn(1)=Va==V1;
18 eqn(2)=Vc-Vb==V2;
19 eqn(3)=-((Vb-Va)/(Zr1))-((Vb)/(Zl))-((Vc)/(Zc))-((Vc)/(Zr2));
20 sol=solve(eqn,Va,Vb,Vc);
21 %Phasor solution will be printed in rectangular and polar form
22 Io=eval(vpa(sol.Vb/Zl,3))
23 fprintf('%0.2f at an angle of %0.2f degrees\n',abs(Io),angle(Io)*(180/pi));

```

#### 4.4 Nodal with Op-Amps

##### EXAMPLE 4.13



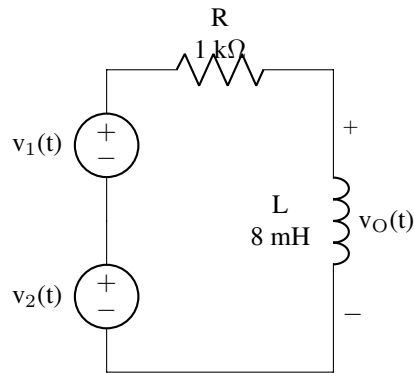
```

1 clear all %Clear all variables in the Workspace
2 close all %Close all open plots
3 clc % clear the Command Window
4 format short eng %Tell MATLAB to report values in fixed, Engineering
   notation
5
6 r1=8e3;%Ohm
7 r2=15e3;%Ohm
8 c1=18e-9;%F
9 c2=12e-9;%F
10 w=1000;%rad/s
11
12 %Move to the phasor-domain
13 Vi=12*exp(j*60*(pi/180));
14
15 Zr1=r1;
16 Zr2=r2;
17 Zc1=inv(j*w*c1);%Ohm
18 Zc2=inv(j*w*c2);%Ohm
19
20 %setup symbolic variables for mesh analysis
21 syms Va Vb Vc Vo
22 eqn(1)=Va==Vi;
23 eqn(2)=Vc==Vo;
24 eqn(3)=( (Va-Vb) / (Zr1) ) - ( (Vb-Vc) / (Zr2) ) - ( (Vb-Vo) / (Zc2) ) ==0;
25 eqn(4)=( (Vb-Vc) / (Zr2) ) - ( (Vc) / (Zc1) );
26 sol=solve(eqn, Va, Vb, Vc, Vo);
27 %Phasor solution will be printed in rectangular and polar form
28 Vo=eval(vpa(sol.Vo, 3))
29 fprintf('%0.2f V at an angle of %0.2f degrees\n', abs(Vo), angle(Vo) * (180/pi));
30
31 Io=eval(vpa((sol.Vo-sol.Vb)/Zc2, 3))
32 fprintf('%0.2f uA at an angle of %0.2f degrees\n', abs(Io*1e6), angle(Io) * (180/
   pi));

```

■ EXAMPLE 4.14





Find  $v_O(t)$  given  $v_1(t) = 5 \cos(130000t - 8.76^\circ) \text{ V}$  and  $v_2(t) = 3 \cos(200000t + 36^\circ) \text{ V}$

```

1 clear all
2 close all
3 clc
4 format short eng
5
6 R=1e3;
7 L=8e-3;
8 w1=130e3;
9 w2=200e3;
10 T1=(2*pi)/w1;
11 T2=(2*pi)/w2;
12
13 t=0:T1/1000:T1;
14 v1=5*exp(j*-8.76*(pi/180));
15 v1_t=abs(v1)*cos(w1*t+angle(v1));
16 v2=3*exp(j*36*(pi/180));
17 v2_t=abs(v2)*cos(w2*t+angle(v2));
18
19 %sub-prob #1
20 Zl=j*w1*L;
21 Zr=R;
22 Vo1=v1*(Zl/(Zl+Zr))
23 abs(Vo1)
24 angle(Vo1)*(180/pi)
25 vo1_t=abs(Vo1)*cos(w1*t+angle(Vo1));
26
27 figure(1)
28 plot(t,v1_t,'LineWidth',3)
29 hold on
30 plot(t,vo1_t,'LineWidth',3,'Color',[1 .5 .5])
31 xlabel('Time (s)')
32 ylabel('Voltage (V)')
33 legend('v1(t)', 'vo1(t)')
34 fig=gcf;
35 set(findall(fig,'-property','FontSize'),'FontSize',12)
36
37 %sub-prob #2
38 Zl=j*w2*L;
39 Zr=R;
40 Vo2=v1*(Zl/(Zl+Zr))
41 abs(Vo2)
42 angle(Vo2)*(180/pi)
43 vo2_t=abs(Vo2)*cos(w2*t+angle(Vo2));

```

```
44
45 figure(2)
46 plot(t,v2_t,'LineWidth',3)
47 hold on
48 plot(t,vo2_t,'LineWidth',3,'Color',[1 .5 .5])
49 xlabel('Time (s)')
50 ylabel('Voltage (V)')
51 legend('v2(t)', 'vo2(t)')
52 fig=gcf;
53 set(findall(fig,'-property','FontSize'),'FontSize',12)
54
55
56 figure(3)
57 plot(t,v1_t+v2_t,'LineWidth',3)
58 hold on
59 plot(t,vo1_t+vo2_t,'LineWidth',3,'Color',[1 .5 .5])
60 xlabel('Time (s)')
61 ylabel('Voltage (V)')
62 legend('v(t)', 'vo(t)')
63
64 fig=gcf;
65 set(findall(fig,'-property','FontSize'),'FontSize',12)
```