

Duality Principle

- Any theorem or postulate in Boolean Algebra remains true if
 - 0 and 1 are swapped and
 - AND and OR are swapped
- Postulates:

| Postulate | Dual Pairs | |
|----------------|-----------------------------------|---|
| Identity | $X+0=X$ | $X \cdot 1=X$ |
| Complements | $X+\bar{X}=1$ | $X \cdot \bar{X}=0$ |
| Commutativity | $X+Y=Y+X$ | $X \cdot Y=Y \cdot X$ |
| Associativity | $(X+Y)+Z=X+(Y+Z)$ | $(X \cdot Y) \cdot Z=X \cdot (Y \cdot Z)$ |
| Distributivity | $X+(Y \cdot Z)=(X+Y) \cdot (X+Z)$ | $X \cdot (Y+Z)=(X \cdot Y)+(X \cdot Z)$ |

Duality Principle

- Theorems

| Theorem | Dual Pairs | |
|---------------|---|---|
| Idempotency | $X+X=X$ | $X \cdot X=X$ |
| Null Elements | $X+1=1$ | $X \cdot 0=0$ |
| Involution | $\overline{\overline{X}}=X$ | $\overline{\overline{X}}=X$ |
| Covering | $X+(X \cdot Y)=X$ | $X \cdot (X+Y)=X$ |
| Covering | $X+(\overline{X} \cdot Y)=X+Y$ | $X \cdot (\overline{X}+Y)=X \cdot Y$ |
| Combining | $(X \cdot Y)+(X \cdot \overline{Y})=X$ | $(X+Y) \cdot (X+\overline{Y})=X$ |
| Combining | $(X \cdot Y)+(X \cdot \overline{Y} \cdot Z)=(X \cdot Y)+(X \cdot Z)$ | $(X+Y) \cdot (X+\overline{Y}+Z)=(X+Y) \cdot (X+Z)$ |
| DeMorgan's | $\overline{X+Y}=\overline{X} \cdot \overline{Y}$ | $\overline{X \cdot Y}=\overline{X}+\overline{Y}$ |
| Consensus | $(X \cdot Y)+(\overline{X} \cdot Z)+(Y \cdot Z)=(X \cdot Y)+(\overline{X} \cdot Z)$ | $(X+Y) \cdot (\overline{X}+Z) \cdot (Y+Z)=(X+Y) \cdot (\overline{X}+Z)$ |
| Shannon's | $f(X, Y, Z)=X \cdot f(1, Y, Z)+\overline{X} \cdot f(0, Y, Z)$ | $f(X, Y, Z)=X+f(0, Y, Z) \cdot \overline{X}+f(1, Y, Z)$ |

