Digital Logic Circuits
'Boolean Algebra'

ELEC2200
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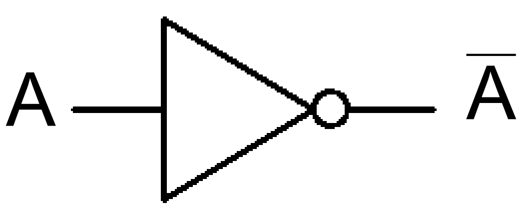
Boolean Algebra

- Developed by George Boole of Bell Labs (1849)
- Originally termed Switching Algebra as it described the math of relay circuits
- Classical Algebra manipulates expressions comprised of real-valued variables and joined by operators such as: Add, Subtract, Multiply, Divide
- Boolean Algebra manipulates expressions comprised of binary-valued variables and joined by operators such as: And, Or, Not
- Other operators can be built from these

Boolean Attributes (Operators)

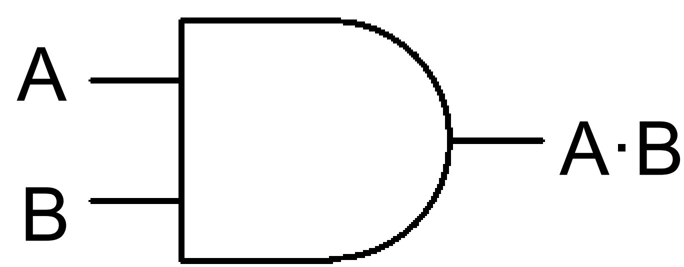
- Complement, Invert, Not
- Two notations: A' and $\overline{A}$

A	$\overline{A}$
0	1
1	0



- And
- Notated as A·B

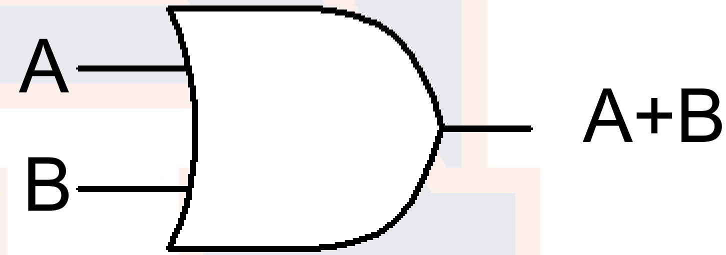
A	B	A·B
0	0	0
0	1	0
1	0	0
1	1	1



Boolean Attributes (Operators)

- Or
- Notated as A+B

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1



Operator Precedence

- Order:
  (1) Parentheses
  (2) Not
  (3) And
  (4) Or
  (5) Left to Right
- When in doubt, use parentheses.

Boolean Postulates

- Identity

$A+0=A$
$A\cdot 1=A$

- Complements

$A+\overline{A}=1$

$A\cdot\overline{A}=0$

A	B	A+0
0	0	0
0	1	1
1	0	1
1	1	1

A	B	A·1
0	0	0
0	1	0
1	0	0
1	1	1

A	B	$A+\overline{A}$
0	0	0
0	1	1
1	0	1
1	1	1

A	B	$A\cdot\overline{A}$
0	0	0
0	1	0
1	0	0
1	1	1

Boolean Postulates

- Commutativity

  $A+B=B+A$

  $A \cdot B=B \cdot A$

A	B	A+B	B+A
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

A	B	A·B	B·A
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

Boolean Postulates

- Associativity

(A+B)+C=A+(B+C)

(A·B)·C=A·(B·C)

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Boolean Postulates

- Associativity

(A+B)+C=A+(B+C)

(A·B)·C=A·(B·C)

A	B	C	A+B
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Boolean Postulates

- Associativity

(A+B)+C=A+(B+C)

(A·B)·C=A·(B·C)

A	B	C	A+B	(A+B)+C
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Boolean Postulates

- Associativity

(A+B)+C=A+(B+C)

(A·B)·C=A·(B·C)

A	B	C	A+B	(A+B)+C	B+C
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

Boolean Postulates

- Associativity

$(A+B)+C=A+(B+C)$

$(A \cdot B) \cdot C=A \cdot (B \cdot C)$

A	B	C	A+B	(A+B)+C	B+C	A+(B+C)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Boolean Postulates

- Associativity

$(A+B)+C=A+(B+C)$

$(A \cdot B) \cdot C=A \cdot (B \cdot C)$

A	B	C	A+B	(A+B)+C	B+C	A+(B+C)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Boolean Postulates

- Associativity

  (A+B)+C=A+(B+C)

  (A·B)·C=A·(B·C)

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Boolean Postulates

- Associativity

  $(A+B)+C=A+(B+C)$

  $(A\cdot B)\cdot C=A\cdot(B\cdot C)$

A	B	C	A·B
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Boolean Postulates

- Associativity

$(A+B)+C=A+(B+C)$

$(A\cdot B)\cdot C=A\cdot(B\cdot C)$

A	B	C	A·B	(A·B)·C
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

Boolean Postulates

- Associativity

$(A+B)+C=A+(B+C)$

$(A \cdot B) \cdot C=A \cdot (B \cdot C)$

A	B	C	A·B	(A·B)·C	B·C
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	1	0	0
1	1	1	1	1	1

Boolean Postulates

- Associativity

  $(A+B)+C=A+(B+C)$

  $(A\cdot B)\cdot C=A\cdot(B\cdot C)$

A	B	C	A·B	(A·B)·C	B·C	A·(B·C)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Boolean Postulates

- Associativity

$(A+B)+C=A+(B+C)$

$(A \cdot B) \cdot C=A \cdot (B \cdot C)$

A	B	C	A·B	(A·B)·C	B·C	A·(B·C)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B \cdot C)=(A+B) \cdot (A+C)$

  $A \cdot (B+C)=(A \cdot B)+(A \cdot C)$

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Boolean Postulates

- Distributivity

  A+(B·C)=(A+B)·(A+C)

  A·(B+C)=(A·B)+(A·C)

A	B	C	A+B
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B\cdot C)=(A+B)\cdot(A+C)$

  $A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

A	B	C	A+B	A+C
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Boolean Postulates

- Distributivity

  A+(B·C)=(A+B)·(A+C)

  A·(B+C)=(A·B)+(A·C)

A	B	C	A+B	A+C	(A+B)·(A+C)
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B \cdot C)=(A+B) \cdot (A+C)$

  $A \cdot (B+C)=(A \cdot B)+(A \cdot C)$

A	B	C	A+B	A+C	(A+B)·(A+C)	B·C
0	0	0	0	0	0	0
0	0	1	0	1	0	0
0	1	0	1	0	0	0
0	1	1	1	1	1	1
1	0	0	1	1	1	0
1	0	1	1	1	1	0
1	1	0	1	1	1	0
1	1	1	1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B\cdot C)=(A+B)\cdot(A+C)$

  $A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

A	B	C	A+B	A+C	(A+B)·(A+C)	B·C	A+(B·C)
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Boolean Postulates

- Distributivity

  A+(B·C)=(A+B)·(A+C)

  A·(B+C)=(A·B)+(A·C)

A	B	C	A+B	A+C	(A+B)·(A+C)	B·C	A+(B·C)
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B\cdot C)=(A+B)\cdot(A+C)$

  $A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Boolean Postulates

- Distributivity

  $A+(B \cdot C)=(A+B) \cdot (A+C)$

  $A \cdot (B+C)=(A \cdot B)+(A \cdot C)$

A	B	C	A·B
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B \cdot C)=(A+B) \cdot (A+C)$

  $A \cdot (B+C)=(A \cdot B)+(A \cdot C)$

A	B	C	A·B	A·C
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B \cdot C)=(A+B) \cdot (A+C)$

  $A \cdot (B+C)=(A \cdot B)+(A \cdot C)$

A	B	C	A·B	A·C	(A·B)+(A·C)
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

Boolean Postulates

- Distributivity

  A+(B·C)=(A+B)·(A+C)

  A·(B+C)=(A·B)+(A·C)

A	B	C	A·B	A·C	(A·B)+(A·C)	B+C
0	0	0	0	0	0	0
0	0	1	0	0	0	1
0	1	0	0	0	0	1
0	1	1	0	0	0	1
1	0	0	0	0	0	0
1	0	1	0	1	1	1
1	1	0	1	0	1	1
1	1	1	1	1	1	1

Boolean Postulates

- Distributivity

  A+(B·C)=(A+B)·(A+C)

  A·(B+C)=(A·B)+(A·C)

A	B	C	A·B	A·C	(A·B)+(A·C)	B+C	A·(B+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	0	1	0
0	1	1	0	0	0	1	0
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	1	1	1
1	1	1	1	1	1	1	1

Boolean Postulates

- Distributivity

  $A+(B\cdot C)=(A+B)\cdot(A+C)$

  $A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

A	B	C	A·B	A·C	(A·B)+(A·C)	B+C	A·(B+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	0	1	0
0	1	1	0	0	0	1	0
1	0	0	0	0	0	0	0
1	0	1	0	1	1	1	1
1	1	0	1	0	1	1	1
1	1	1	1	1	1	1	1

Boolean Theorems

- Idempotency

  A+A=A

  A·A=A

- Null Elements

  A+1=1

  A·0=0

- Involution

  $\overline{\overline{A}}$=(A')'=A

A	A+A	A·A	A+1	A·0	$\overline{A}$	$\overline{\overline{A}}$
0	0	0	1	0	1	0
1	1	1	1	0	0	1

Boolean Theorems

- Absorption (Covering)

  $A \cdot (A+B) = A$

  $A + (A \cdot B) = A$

A	B	A+B	A·B	A·(A+B)	A+(A·B)
0	0	0	0	0	0
0	1	1	0	0	0
1	0	1	0	1	1
1	1	1	1	1	1

Boolean Theorems

- Absorption (Covering)

  $A \cdot (\overline{A}+B)=A \cdot B$

  $A+(\overline{A} \cdot B)=A+B$

A	B	$\overline{A}$	$\overline{A}+B$	$A \cdot (\overline{A}+B)$	$A \cdot B$	$\overline{A} \cdot B$	$A+(\overline{A} \cdot B)$	$A+B$
0	0	1	0	0	0	0	0	0
0	1	1	1	0	0	1	1	1
1	0	0	0	0	0	0	1	1
1	1	0	1	1	1	0	1	1

Boolean Theorems

- Absorption (Combining)

$(A+B)\cdot(A+\overline{B})=A$

$(A\cdot B)+(A\cdot\overline{B})=A$

A	B	$\overline{B}$	A+B	$A+\overline{B}$	$(A+B)\cdot(A+\overline{B})$	$A\cdot B$	$A\cdot\overline{B}$	$(A\cdot B)+(A\cdot\overline{B})$
0	0	1	0	1	0	0	0	0
0	1	0	1	0	0	0	0	0
1	0	1	1	1	1	0	1	1
1	1	0	1	1	1	1	0	1

Boolean Theorems

- Absorption (Combining)

  $(A \cdot B)+(A \cdot \overline{B} \cdot C)=(A \cdot B)+(A \cdot C)$

  $(A+B) \cdot (A+\overline{B}+C)=(A+B) \cdot (A+C)$

A	B	C	$\overline{B}$	A·B	A·$\overline{B}$·C	A·C	(A·B)+(A·$\overline{B}$·C)	(A·B)+(A·C)
0	0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	1	0	1	1	1	1
1	1	0	0	1	0	0	1	1
1	1	1	0	1	0	1	1	1

Boolean Theorems

- Absorption (Combining)

$(A \cdot B)+(A \cdot \overline{B} \cdot C)=(A \cdot B)+(A \cdot C)$

$(A+B) \cdot (A+\overline{B}+C)=(A+B) \cdot (A+C)$

A	B	C	$\overline{B}$	A+B	$A+\overline{B}+C$	A+C	$(A+B) \cdot (A+\overline{B}+C)$	$(A+B) \cdot (A+C)$
0	0	0	1	0	1	0	0	0
0	0	1	1	0	1	1	0	0
0	1	0	0	1	0	0	0	0
0	1	1	0	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1
1	1	1	0	1	1	1	1	1

Boolean Theorems

- DeMorgan's Theorem

$$\overline{A+B}=\overline{A}\cdot\overline{B}$$

$$\overline{A\cdot B}=\overline{A}+\overline{B}$$

- Break the bar and change the operator
- Connect the bar and change the operator
- Expands to include more terms(Generalized DeMorgan's):

$$\overline{A+B+C+\ldots}=\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\ldots$$

$$\overline{A\cdot B\cdot C\cdot\ldots}=\overline{A}+\overline{B}+\overline{C}+\ldots$$

Boolean Theorems

- DeMorgan's Theorem

$$\overline{A+B}=\overline{A}\cdot\overline{B}$$

$$\overline{A\cdot B}=\overline{A}+\overline{B}$$

A	B	A+B	A·B	$\overline{A+B}$	$\overline{A\cdot B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$\overline{A}+\overline{B}$
0	0	0	0	1	1	1	1	1	1
0	1	1	0	0	1	1	0	0	1
1	0	1	0	0	1	0	1	0	1
1	1	1	1	0	0	0	0	0	0

Boolean Theorems

- Consensus Theorem
- $(A \cdot B)+(\overline{A} \cdot C)+(B \cdot C)=(A \cdot B)+(\overline{A} \cdot C)$
- $(A+B) \cdot (\overline{A}+C) \cdot (B+C)=(A+B) \cdot (\overline{A}+C)$

A	B	C	$\overline{A}$	$A \cdot B$	$\overline{A} \cdot C$	$B \cdot C$	$(A \cdot B)+(\overline{A} \cdot C)+(B \cdot C)$	$(A \cdot B)+(\overline{A} \cdot C)$
0	0	0	1	0	0	0	0	0
0	0	1	1	0	1	0	1	1
0	1	0	1	0	0	0	0	0
0	1	1	1	0	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	1	0	0	1	1
1	1	1	0	1	1	1	1	1

Boolean Theorems

- Consensus Theorem
- $(A \cdot B)+(\overline{A} \cdot C)+(B \cdot C)=(A \cdot B)+(\overline{A} \cdot C)$
- $(A+B) \cdot (\overline{A}+C) \cdot (B+C)=(A+B) \cdot (\overline{A}+C)$

A	B	C	$\overline{A}$	A+B	$\overline{A}$+C	B+C	$(A+B) \cdot (\overline{A}+C) \cdot (B+C)$	$(A+B) \cdot (\overline{A}+C)$
0	0	0	1	0	1	0	0	0
0	0	1	1	0	1	1	0	0
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	0	0	0
1	0	1	0	1	1	1	1	1
1	1	0	0	1	0	1	0	0
1	1	1	0	1	1	1	1	1

Boolean Theorems

- Shannon's Expansion

$$f(A,B,C)=(A\cdot f(1,B,C))+(\overline{A}\cdot f(0,B,C))$$

$$f(A,B,C)=(A+f(0,B,C))\cdot(\overline{A}+f(1,B,C))$$

- Example:

$$f(A,B,C)=A\cdot B\cdot C+A\cdot\overline{B}\cdot C+\overline{A}\cdot\overline{B}\cdot C$$

$$f(A,B,C)=A\cdot f(1,B,C)+\overline{A}\cdot f(0,B,C)$$

$$f(1,B,C)=B\cdot C+\overline{B}\cdot C$$

$$f(0,B,C)=\overline{B}\cdot C$$

$$f(A,B,C)=A\cdot(B\cdot C+\overline{B}\cdot C)+\overline{A}\cdot(\overline{B}\cdot C)$$

- Important for circuit minimization

Duality Principle

- Any theorem or postulate in Boolean Algebra remains true if
  - 0 and 1 are swapped and
  - AND and OR are swapped
- Postulates:

Postulate	Dual Pairs	
Identity	$A+0=A$	$A\cdot 1=A$
Complements	$A+\overline{A}=1$	$A\cdot\overline{A}=0$
Commutativity	$A+B=B+A$	$A\cdot B=B\cdot A$
Associativity	$(A+B)+C=A+(B+C)$	$(A\cdot B)\cdot C=A\cdot(B\cdot C)$
Distributivity	$A+(B\cdot C)=(A+B)\cdot(A+C)$	$A\cdot(B+C)=(A\cdot B)+(A\cdot C)$

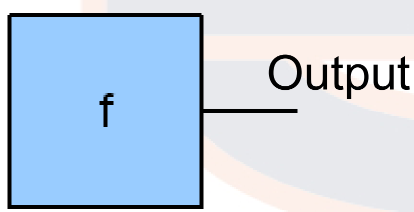
Duality Principle

- Theorems

Theorem	Dual Pairs	
Idempotency	$A+A=A$	$A\cdot A=A$
Null Elements	$A+1=1$	$A\cdot 0=0$
Involution	$\overline{\overline{A}}=A$	$\overline{\overline{A}}=A$
Covering	$A+(A\cdot B)=A$	$A\cdot(A+B)=A$
Covering	$A+(\overline{A}\cdot B)=A+B$	$A\cdot(\overline{A}+B)=A\cdot B$
Combining	$(A\cdot B)+(A\cdot\overline{B})=A$	$(A+B)\cdot(A+\overline{B})=A$
Combining	$(A\cdot B)+(A\cdot\overline{B}\cdot C)=(A\cdot B)+(A\cdot C)$	$(A+B)\cdot(A+\overline{B}+C)=(A+B)\cdot(A+C)$
DeMorgan's	$\overline{A+B}=\overline{A}\cdot\overline{B}$	$\overline{A\cdot B}=\overline{A}+\overline{B}$
Consensus	$(A\cdot B)+(\overline{A}\cdot C)+(B\cdot C)=(A\cdot B)+(\overline{A}\cdot C)$	$(A+B)\cdot(\overline{A}+C)\cdot(B+C)=(A+B)\cdot(\overline{A}+C)$
Shannon's	$f(A,B,C)=A\cdot f(1,B,C)+\overline{A}\cdot f(0,B,C)$	$f(A,B,C)=A+f(0,B,C)\cdot\overline{A}+f(1,B,C)$

Switching Functions

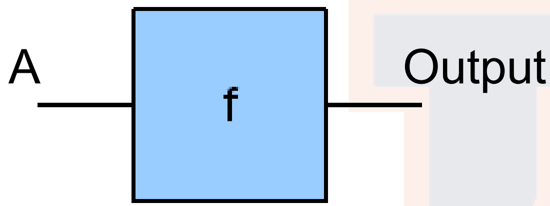
- We've shown proof that two expressions are equivalent by testing every possible input
- We can also generate every expression given the # of inputs, n
- The simplest (trivial) case, n=0, or no inputs:



	f0	f1
	0	1

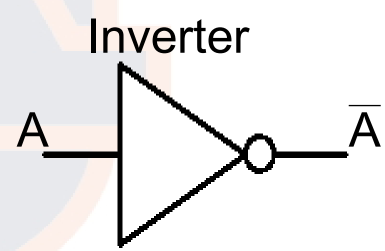
Switching Functions

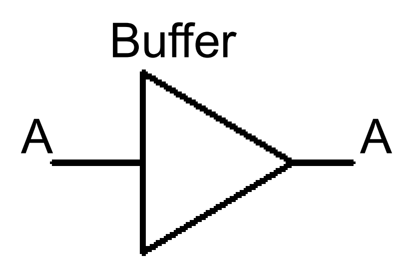
- 1 input, n=1



A	f0	f1	f2	f3
0	0	1	0	1
1	0	0	1	1

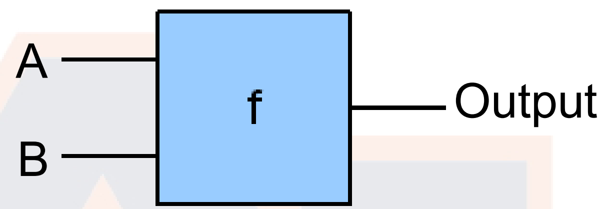
Function	Expression	Description
f0	0	Logic 0
f1	$\overline{A}$	Inverter
f2	A	Buffer
f3	1	Logic 1





Switching Functions

- 2 inputs, n=2

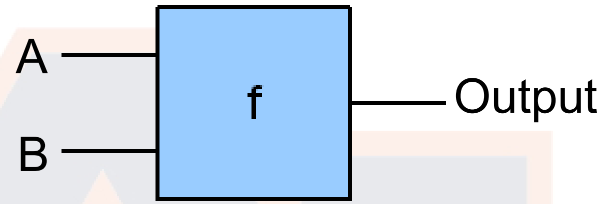


A	B	f0	f1	f2	f3	f4	f5	f6	f7	f8	f9	fA	fB	fC	fD	fE	fF
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

- Looking closely, we've seen some of these before

Switching Functions

- 2 inputs, n=2

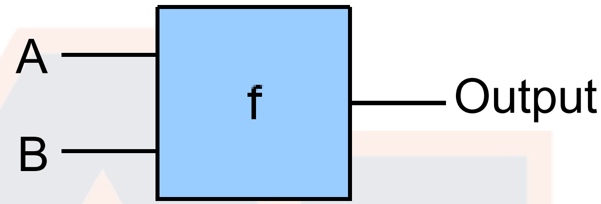


A	B	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_A	f_B	f_C	f_D	f_E	f_F
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

- These are the two trivial logic sources
  - $f_0 = 0$
  - $f_F = 1$

Switching Functions

- 2 inputs, n=2

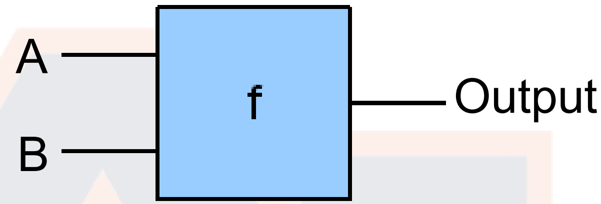


A	B	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_A	f_B	f_C	f_D	f_E	f_F
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

- f_C and f_A are buffers for A and B respectively
- f_3 and f_5 are inverters for A and B respectively

Switching Functions

- 2 inputs, n=2

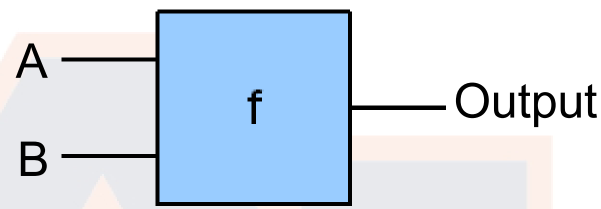


A	B	f 0	f 1	f 2	f 3	f 4	f 5	f 6	f 7	f 8	f 9	f A	f B	f C	f D	f E	f F
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

- f_E is A+B
- f_8 is A·B

Switching Functions

- 2 inputs, n=2

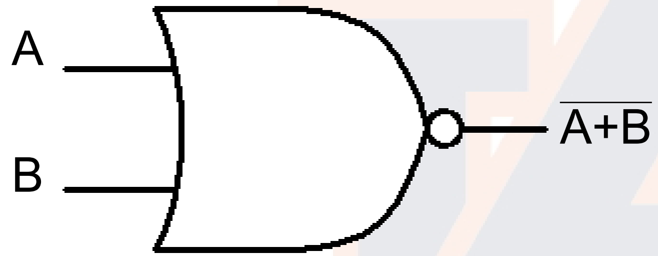


A	B	f0	f1	f2	f3	f4	f5	f6	f7	f8	f9	fA	fB	fC	fD	fE	fF
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

- f_1 is $\overline{A+B}$, NOR
- f_7 is $\overline{A \cdot B}$, NAND

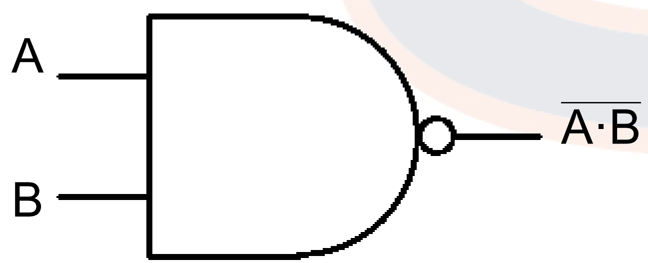
New Operators

- NOR



A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

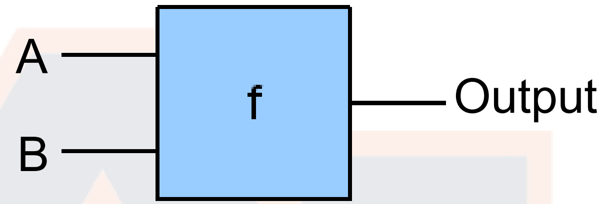
- NAND



A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

Switching Functions

- 2 inputs, n=2

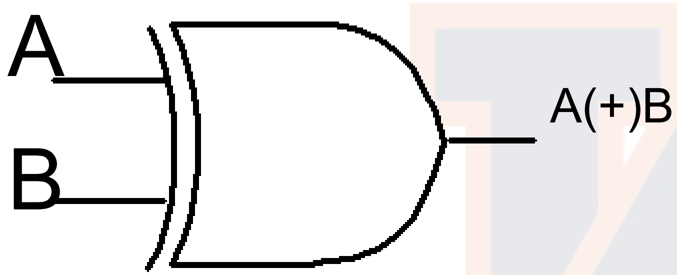


A	B	f0	f1	f2	f3	f4	f5	f6	f7	f8	f9	fA	fB	fC	fD	fE	fF
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

- f_6, Exclusive OR, XOR
- f_9, Exclusive NOR, XNOR

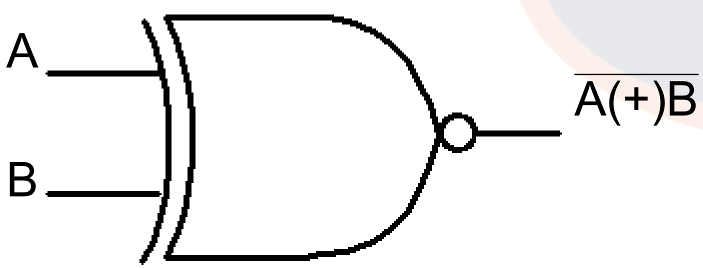
New Operators

- XOR



A	B	A(+)B
0	0	0
0	1	1
1	0	1
1	1	0

- XNOR



A	B	$\overline{A(+)B}$
0	0	1
0	1	0
1	0	0
1	1	1

Switching Function, n=2

Function	Expression	Description
f0	0	Logic 0
f1	$\overline{A+B}$	NOR
f2	$\overline{A}\cdot B$	
f3	$\overline{A}$	Inverter A
f4	$A\cdot\overline{B}$	
f5	$\overline{B}$	Inverter B
f6	A(+)B	XOR
f7	$\overline{A\cdot B}$	NAND
f8	A·B	AND
f9	$\overline{A(+)B}$	XNOR
fA	B	Buffer B
fB	$\overline{A}+B$	
fC	A	Buffer A
fD	$A+\overline{B}$	
fE	A+B	OR
fF	1	Logic 1

Switching Functions

- How many possible functions given # of inputs, n?
  - n=0, 2 functions
  - n=1, 4 functions
  - n=2, 16 functions
- In general there are 2^{2^n} functions
- So for n=3 there are 256 possible functions